Kuwait University
College of Engineering and Petroleum

جامعة الكويت
KUWAIT UNIVERSITY

ME319 MECHATRONICS

Part I: The Brains – Microcontrollers, Software and Digital Logic

Lecture 2: Digital Arithmetic

Spring 2021

Ali AlSaibie

Lecture Plan

- Objectives:
  - Review the basic numeral systems used in computing
  - Learn how to represent and convert numbers from and to a numeral system
  - Review the representation of negative numbers
- Reading:
  - Chapter 3 & 4, Basic Microprocessors and the 6800, Ron Bishop

- The numeral system we normally use is the decimal numeral system.
- Decimal numbers are represented using base **10**

$$645 = 6 \times \underbrace{10}_{Base\ 10}{}^{2} + 4 \times 10^1 + 5 \times 10^0$$

  - Every digit can be one of the ten numbers from **0** to **9**
- The same applies to decimal point fractions

$$6.45 = 6 \times 10^0 + 4 \times 10^{-1} + 5 \times 10^{-2}$$

- In the world of computers, transistors live in binary states
  - **Bi** means “two”; transistors are either **“on”** or **“off”**
  - The two states are denoted by the binary digits **1** and **0**

Number	**Decimal**	**Binary**
0	0	0000 0000
2^0	1	0000 0001
2^1	2	0000 0010
2^2	4	0000 0100
2^3	8	0000 1000
2^4	16	0001 0000
⋮	⋮	⋮
2^7	128	1000 0000

جامعة الكويت
KUWAIT UNIVERSITY

- A decimal number can be converted to binary by division
- Convert the decimal number **134** to binary

$134{:}\,2 = 67 + 0$ (LSB)
$67{:}\,2 = 33 + 1$
$33{:}\,2 = 16 + 1$
$16{:}\,2 = 8 + 0$
$8{:}\,2 = 4 + 0$
$4{:}\,2 = 2 + 0$
$2{:}\,2 = 1 + 0$
$1{:}\,2 = 0 + 1$ (MSB)

- Decimal 134 = Binary 1000 0110
- Binary numbers can be denoted by a leading 0b: $0b1000\ 0110$

جامعة الكويت
KUWAIT UNIVERSITY

Binary (Base 2) to Decimal (Base 10)

- To convert from binary to decimal, each bit is multiplied by 2 to the power n, where n is the order of the digit, then the result is the sum of the terms.
- Convert the binary number $0101\ 0111$ to decimal

$$0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$
$$= 0 \cdot 128 + 1 \cdot 64 + 0 \cdot 32 + 1 \cdot 16 + 0 \cdot 8 + 1 \cdot 4 + 1 \cdot 2 + 1 \cdot 1$$
$$= 0 + 64 + 0 + 16 + 0 + 4 + 2 + 1$$
$$= 87$$

جامعة الكويت
KUWAIT UNIVERSITY

Bits and Bytes

- A series of 8 bits can have $2^8 = 256$ different combinations (distinct values).
- A series of n-bits can have 2^n different combinations.
- In other words, an 8-bit binary number can take any value between 0 to 255

$$1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 255$$

$$0 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = 0$$

- A series of 8-bits is called a **Byte**
- A series of 4-bits is called a **nibble**

Convert the following numbers to binary representation

a. 10_{10}

b. 89_{10}

جامعة الكويت
KUWAIT UNIVERSITY

Convert the following binary numbers to decimal representation

Example 2

a. 1010 0101

b. 0000 1000

جامعة الكويت
KUWAIT UNIVERSITY

- If not specified, a binary number represents an ***unsigned*** (positive) number.
  - A byte can express the range of unsigned numbers from 0 to 255
  - Two bytes can express the range of unsigned number from 0 to $2^{16} - 1$
    - From 0 to 65,535
- To express a negative decimal number in binary, the data type must be treated as a ***signed*** number.
- Signed numbers lose one bit (MSB) to denote the sign. Signed numbers can use a one's complement or two's complement representation
  - With one's complement, an 8-bit signed number can take any value between -127 to 127
  - With two's complement, an 8-bit signed number can take any value between -128 to 127

Example 3

Answer the following questions:

a. What is the range of values that can be represented by an 7-bit unsigned number?

b. What is the range of values that can be represented by a 10-bit signed number?

c. How many bytes are required to store the decimal number -12304?

جامعة الكويت
KUWAIT UNIVERSITY

One's Complement

- A binary numbers complement denotes the opposite sign equivalent of that number
- The one's complement form of a binary number is the **bitwise NOT** of the number (flipping the bit values)
- The decimal number 50_{10} is represented as $0011\,0010$, its one's complement is $1100\,1101 = -50_{10}$

$$\sim 0011\,0010 = 1100\,1101$$

- The number 0_{10} is expressed as $0000\,0000$ or by its one's complement: $1111\,1111$

$$\sim 0000\,0000 = 1111\,1111$$

  - This redundant representation is resolved by using the two's complement representation

جامعة الكويت
KUWAIT UNIVERSITY

- With two’s complement representation, there is only one zero
- For a byte $0_{10} = 0000\,0000$
- $1111\,1111$ represents -1_{10} in two’s complement, as opposed to **0** in one’s complement representation
- $1000\,0000$ represents -128_{10}
- The number 21_{10} is represented as $0001\,0101$, its one’s complement is the bitwise NOT + 1
- $-21_{10} = \sim 0001\,0101 + 1 = 1110\,1010 + 1 = 1110\,1011$
- Two’s complement is the standard way of representing signed values

جامعة الكويت
KUWAIT UNIVERSITY

- Binary numbers can be added/subtracted by carrying over the extra
- Add the numbers 181 and 53 in binary

	1	1		1		1		
1	0	1	1	0	1	0	1	
0	0	1	1	0	1	0	1	+
1	**1**	**1**	**0**	**1**	**0**	**1**	**0**	

- Answer is $234_{10} = 0b1110\,1010$
- Note that the above are treated as unsigned numbers
- Signed numbers can be added the same way (by using two's complements)

جامعة الكويت
KUWAIT UNIVERSITY

Example 4

Add the following two numbers by using their 8-bit binary representations

$a.\quad 100 + 23$

$b.\quad 100 - 23$

جامعة الكويت
KUWAIT UNIVERSITY

- There is a more compact way to represent numbers rather than binary
- Every nibble can take $2^4 = 16$ different values. A hexadecimal digit represents a nibble.
- The 16 hexadecimal digits are: $0,1,2,3,4,5,6,7,8,9, A, B, C, D, E, F$
    - 10 numerical digits + 6 alphabetical letters
- A 32-bit number can be represented by 8 hexadecimal digits
- A hexadecimal number is denoted by a leading 0x
- $0\text{b}1111\ 0101\ 0010\ 0110 = 0\text{xF}526$

Perform the following numeral system conversions

Example 5

a. 0xFFA1 to binary

b. 0x0BA1 to decimal

c. 350_{10} to hexadecimal

جامعة الكويت
KUWAIT UNIVERSITY

Example 6

Add the following numbers using their 16-bit hexadecimal representations

a. $0xC8 + 0xF0$

b. $0x1E + 0xBB$

جامعة الكويت
KUWAIT UNIVERSITY