

# Kuwait University

## College of Engineering and Petroleum



جامعة الكويت  
KUWAIT UNIVERSITY

## **ME319 MECHATRONICS**

PART IV: THE MUSCLES – ACTUATORS

LECTURE 1: DC MOTORS

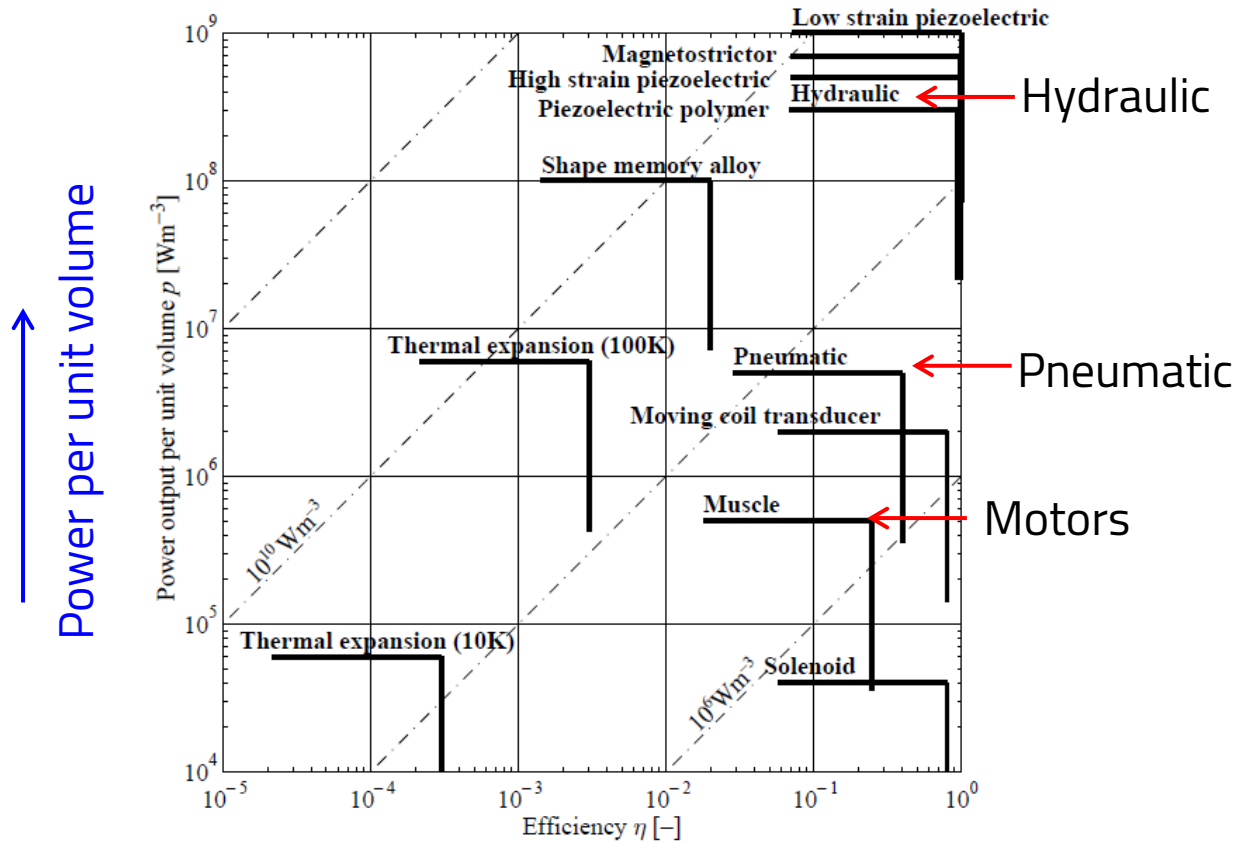
Spring 2021

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# Electric Actuators

- Advantages of electric actuators
- Clean (do not require fluids, oil, etc.)
- Require no extra equipment  
(no need for pressure tanks, etc.)
- Can operate indoors (no emissions)
- Can be made small economically
- Disadvantages of electric actuators
- Low power-to-size ratio



Taken from: Huber, J., Fleck, N., Ashby, M., "The Selection of Mechanical Actuators Based on Performance Indices," Proc. R. Soc. Lond. A, Vol. 453, 1997, pp. 2185-2205.



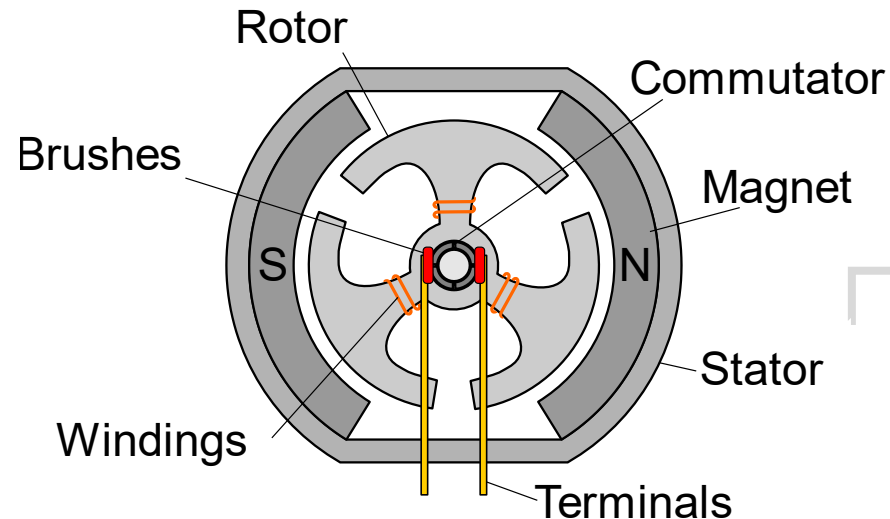
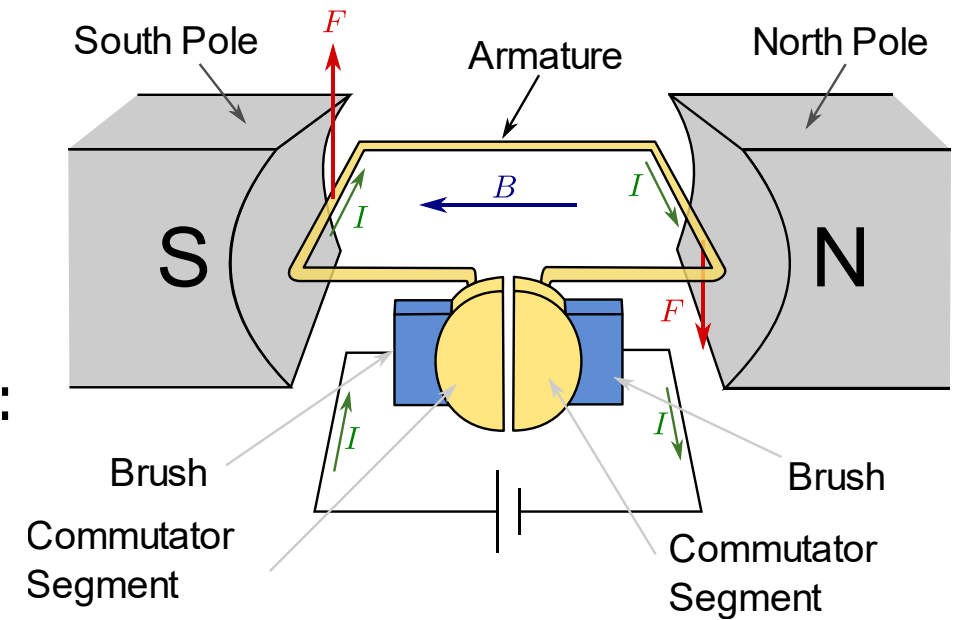
# Electric Actuators

- Most commonly: Motors
  - Electric solenoids can be classified as electric actuators too
- Motors
  - Many different types, names, technologies
  - No uniform way to classify them
  - More logical classify by group rather than type (Operating principle)
- Motor *Groups* (Hughes' Electric Motors and Drives, 2013):
  1. Conventional DC Motors
  2. Induction Motors
  3. Synchronous and Brushless Permanent Magnet Motors
  4. Stepping and Switched-Reluctance



# Conventional DC Motors

- a.k.a Brushed DC Motors
  - The ones you find in most toys
  - Simple Operation
    - Supply voltage across them, they move.
- Brushed motors composed of two components:
  - Stator (remains stationary)
  - Rotor (turns, coupled to shaft)
- Lorentz's Law  $\vec{F} = \vec{I} \times \vec{B}$ 
  - Wire coil runs along back end of armature to generate B field
  - Commutator used to change direction of current flow as armature rotates



# Conventional DC Motor Operation

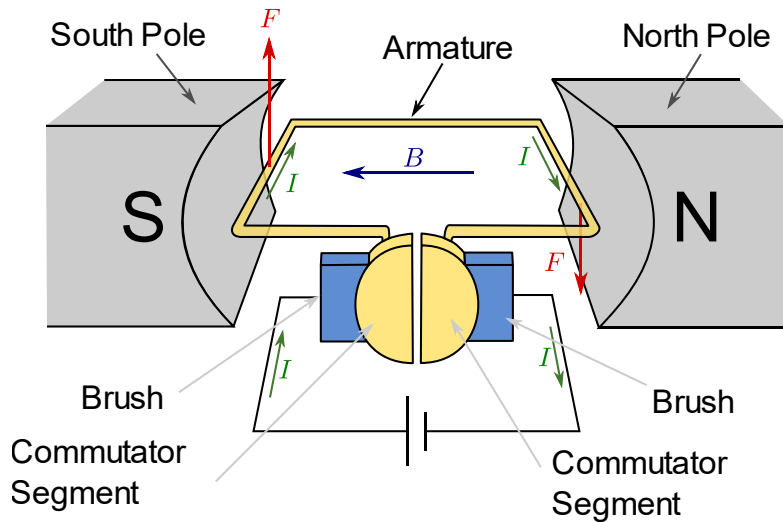
- Commutator must be composed of at least two segments
  - Motors on previous page had 3-piece commutator (left) and 2-piece commutator (right)
- As motor turns, angle of energized coil with respect to magnet changes

$$\vec{F} = \vec{I} \times \vec{B} \longrightarrow$$

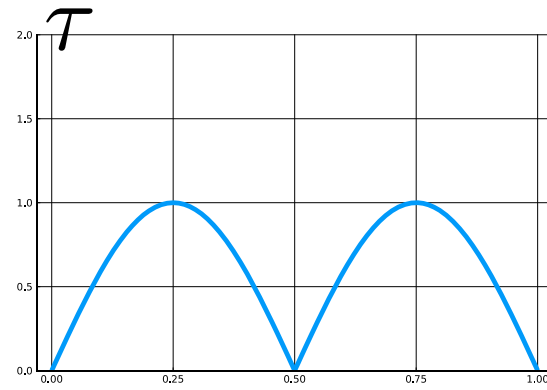
Torque is function of sine of angle between B field and armature angle



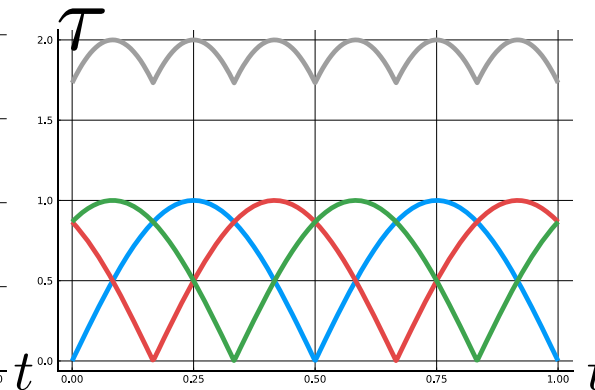
Torque is not smooth using 2-piece commutator



*Torque vs angle for 2-piece comm.*



*Torque vs angle for 6-piece comm.*



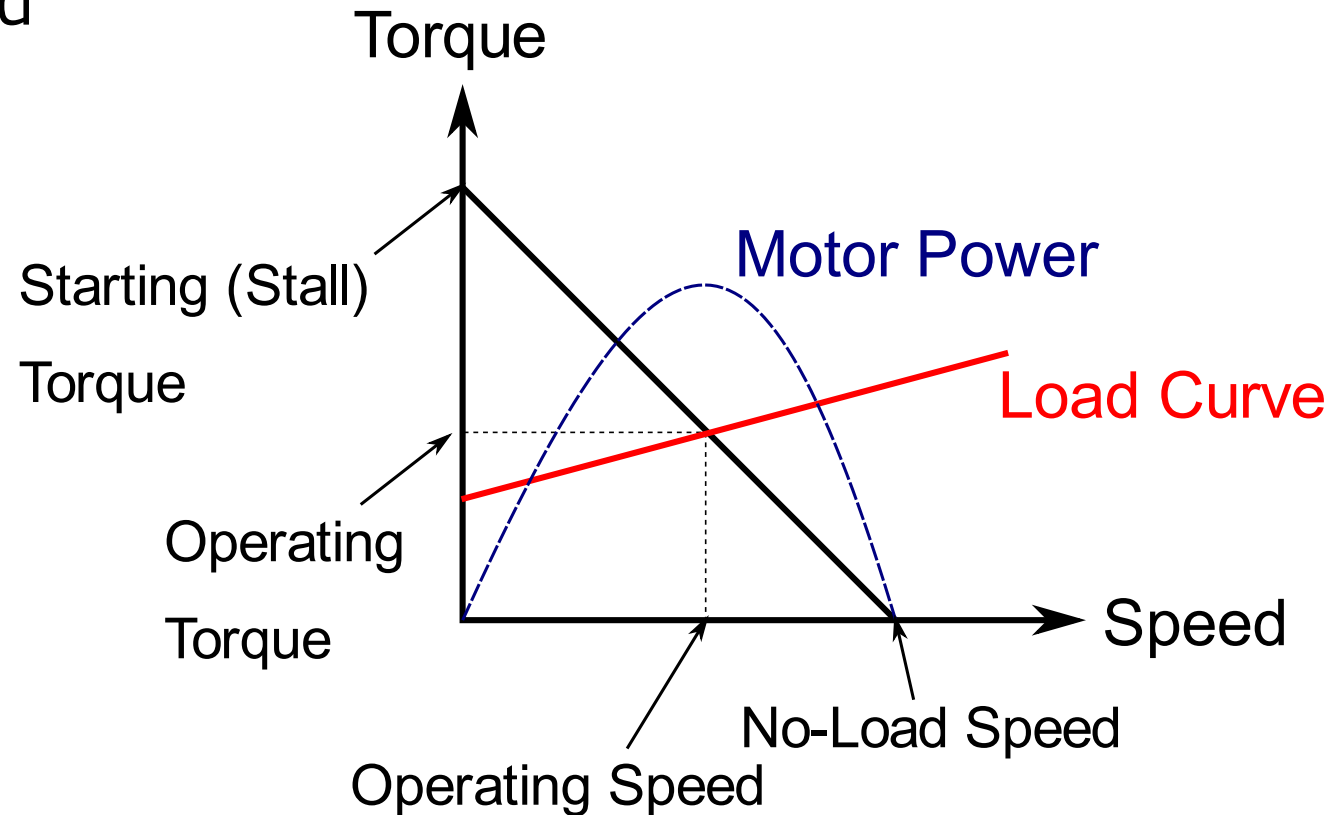
# Brushed DC Motor

- [Video](#)



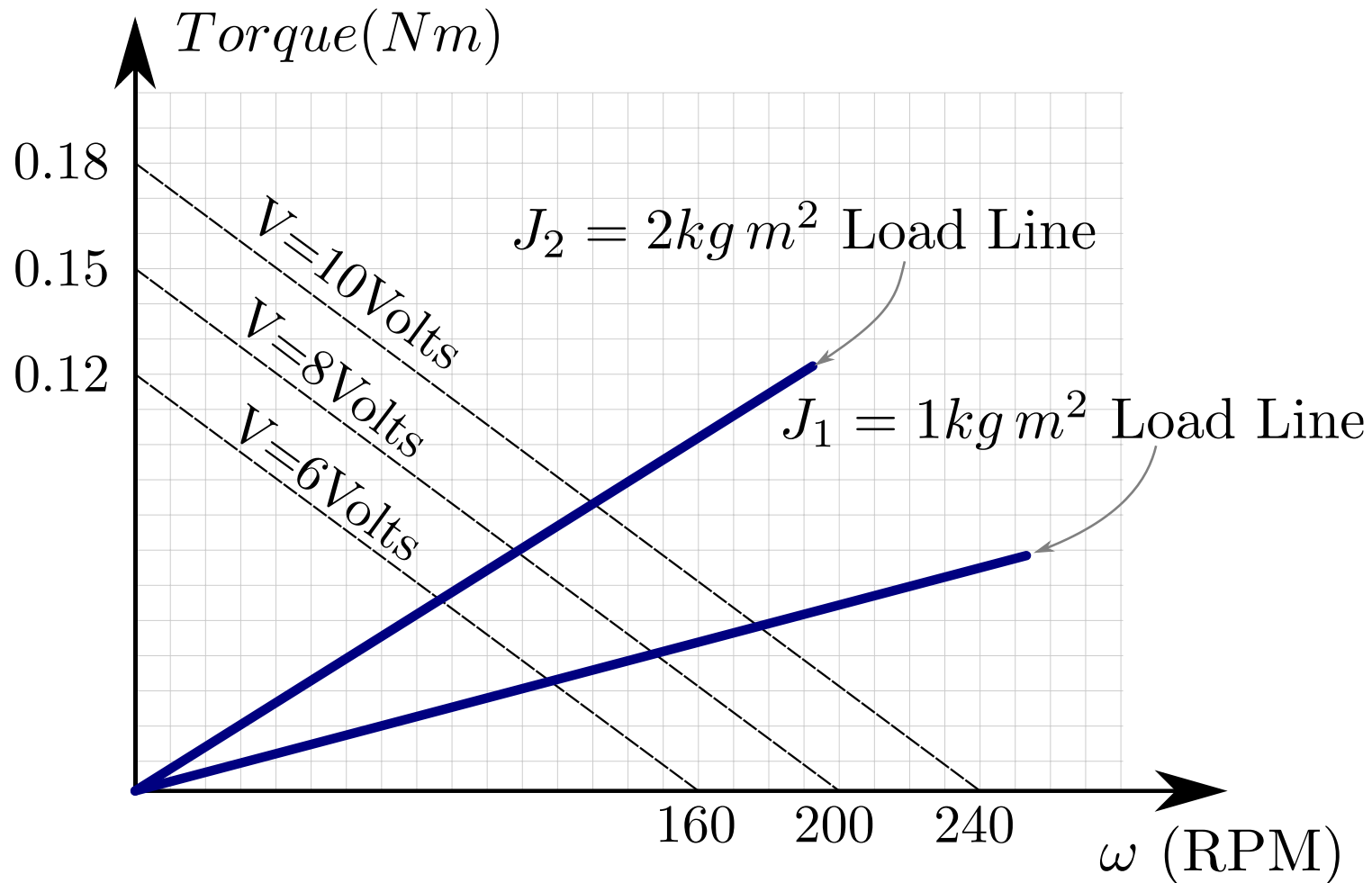
# Conventional PM DC Motor Analysis

- Torque-speed curve defined by two parameters (Assuming linear torque-speed curve)
  - Starting (stall) torque – max torque when speed is zero
  - No-load speed



# Torque – Speed Curve – conventional DC Motor

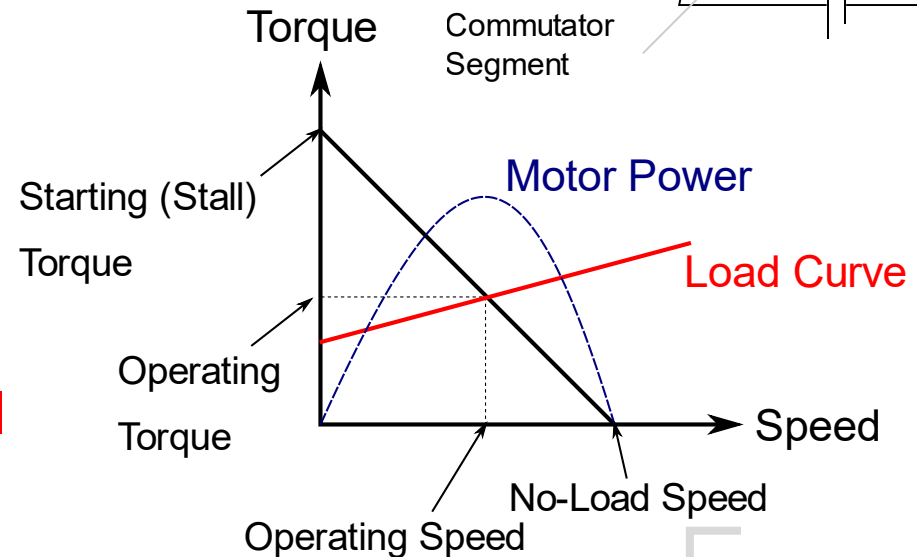
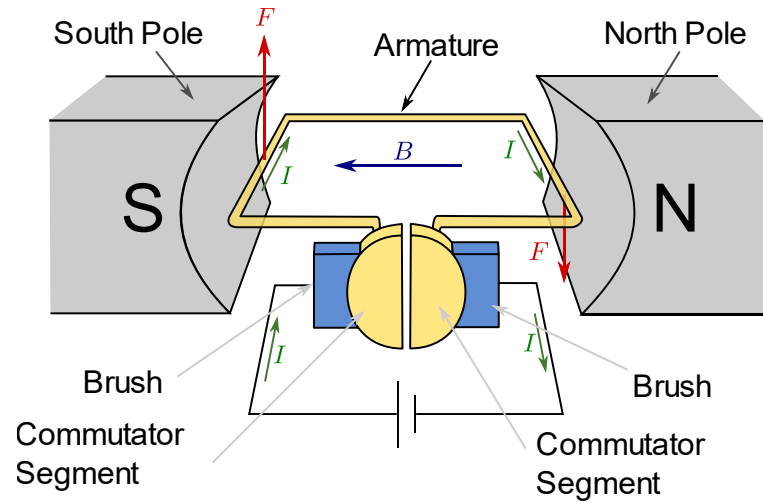
- Note that for a given load, we get the load line, which relates the output torque to various voltage values, and the corresponding speed





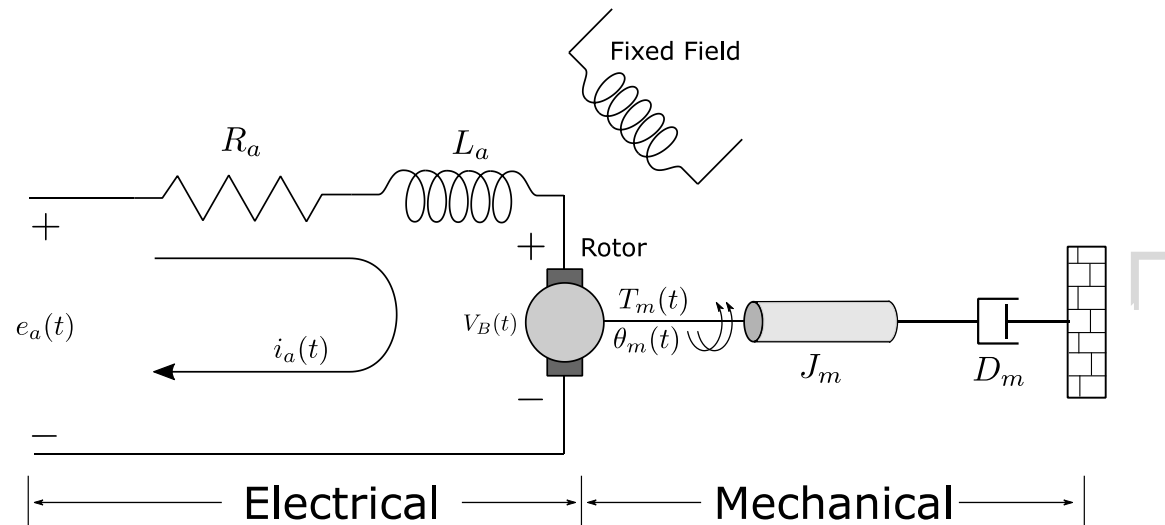
# Conventional PM DC Motor Analysis

- As motor rotates, back EMF generated due to rotation of coil within magnetic field reduces voltage across motor leads and thus current through motor
- This is why torque decreases with speed
- Torque continues to decrease as  $\omega$  increases until torque is zero at maximum speed
- Motor delivers maximum power when it reaches half of its no load speed:  $P = T\omega$  ( $T = \text{Torque}$ )
- When motor drives a load, its operating speed will be where load torque equals motor torque
- If load torque increases linearly with speed, operating speed of motor will increase linearly with increase in supply voltage



# DC Motor Model

- The DC Motor is modeled as a circuit + rotational mechanical system
- On the electrical side, we apply voltage across the coils which have resistance and inductance, modeled as  $R_a$  and  $L_a$ .
  - $R_a I_a(s) + L_a s I_a(s) + V_B(s) = E_a(s)$  (eq. 1)
- The motor is represented as a back-emf voltage in the circuit,  $V_B(t)$ .
  - $V_B = K_B \dot{\theta}_m \Leftrightarrow V_B(s) = K_B s \Theta_m(s)$  (eq. 2)
  - $K_B$ : back-emf constant

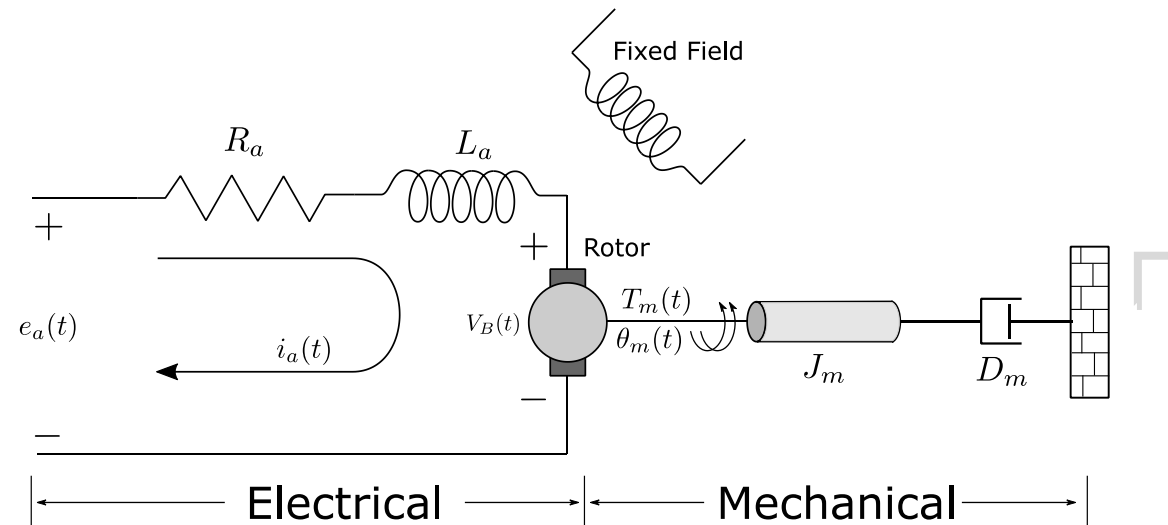


# DC Motor Model

- The Fixed Field represents the permanent magnets' field.
- The torque developed by the motor is proportional to the field current.
  - $T_m(s) = K_t I_a(s)$  (eq. 3),  $K_t$ : motor torque constant

- Combining equations 1 to 3

- $$\frac{(R_a + L_a s) T_m(s)}{K_t} + K_B s \Theta_m(s) = E_a(s) \quad (\text{eq. 4})$$



# DC Motor Model

- On the mechanical side, the motor itself has an inertia  $J_m$ , that rotates with angular velocity  $\theta_m$ , in addition to mechanical bearing friction (viscous damping)  $D_m$

- $T_m(s) = (J_m s^2 + D_m s)\Theta_m(s)$  (eq. 5)

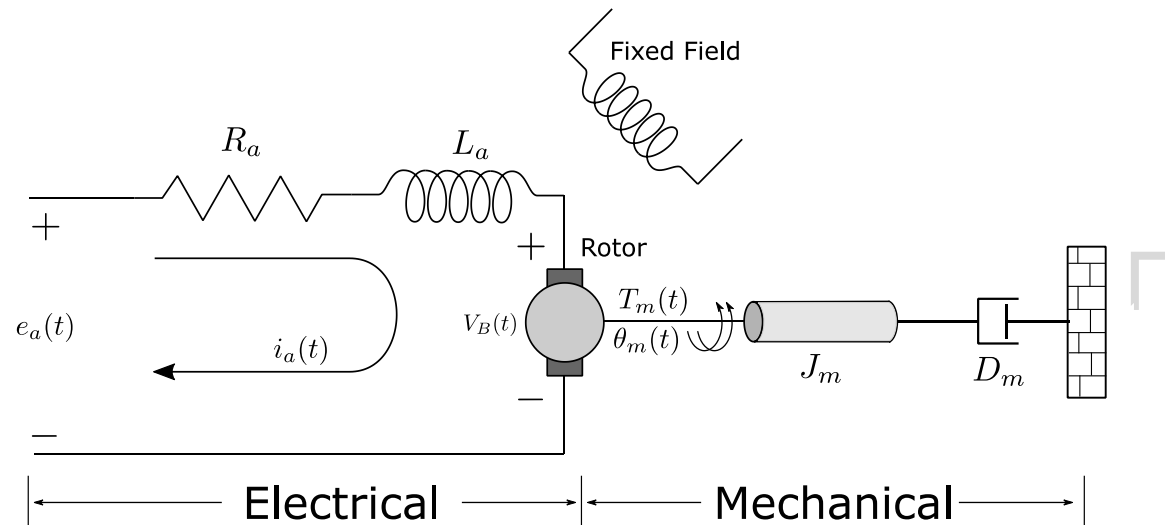
- Combining equations 4 & 5, and rearranging to express  $\frac{\Theta_m(s)}{E_m(s)}$ , ignoring  $L_a$  as  $R_a/L_a \gg 1$

- $\frac{\Theta_m(s)}{E_m(s)} = \frac{K_t/(R_a J_m)}{s[s + \frac{1}{J_m}(D_m + \frac{K_t K_B}{R_a})]}$  (eq. 6)  $= \frac{K}{s(s + \alpha)}$

- The constants  $K, \alpha$

- $K = K_t/(R_a J_m)$

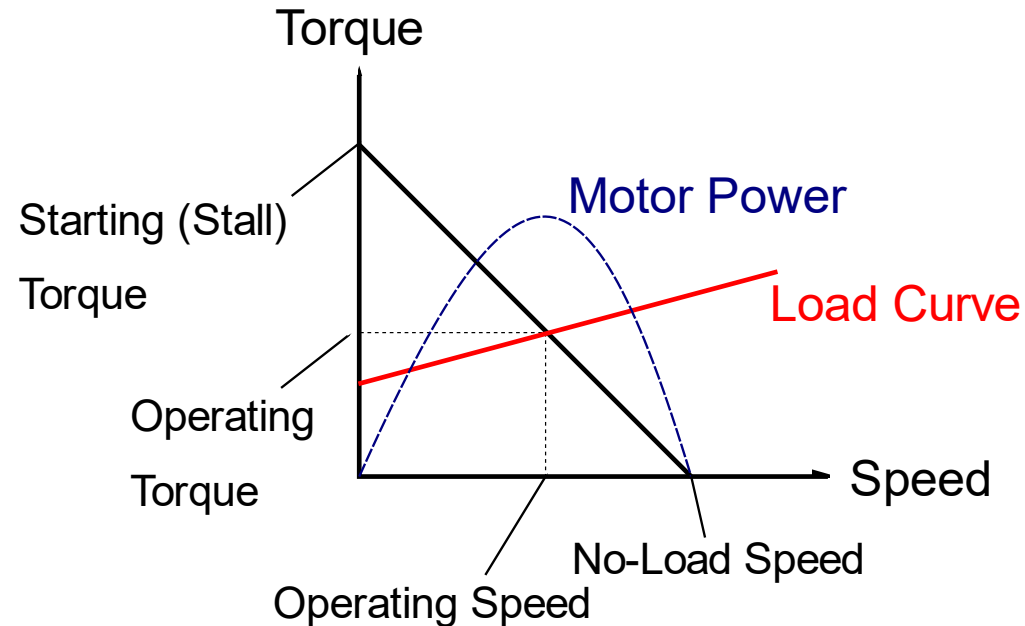
- $\alpha = \frac{1}{J_m} (D_m + \frac{K_t K_B}{R_a})$



# DC Motor Model

- How do we find the constants' values?
- A dynamometer can be used to generate a profile of the motor
- A dynamometer is a test bench for motors, allows for changing mechanical load, changing supplied voltage and measuring generated torque and current consumed. The generated profile is a torque-speed curve.

**Ideal** Steady-State Torque-Speed Curve for a Brushed DC Motor



*Note: If you search for Torque-Speed curves you will get different shaped curves, those are for different types of motors or different assumptions made or different operating conditions, but the key characteristics above apply.*



# DC Motor – Profiling Steady-State Characteristics

- From equation 4, if we consider the steady-state response of the motor, we can simplify by setting inductance  $L_a = 0$ , we get

- $$\frac{R_a}{K_t} T_m(s) + K_B s \Theta_m(s) = E_a(s) \quad (\text{eq. 6})$$

- Taking  $\mathcal{L}^{-1}(\text{eq. 6})$  and rearranging

- $$T_m(t) = -\frac{K_B K_t}{R_a} \omega_m(t) + \frac{K_t}{R_a} e_a(t) \quad (\text{eq. 7})$$

- Equation 7 matches the Torque-Speed Curve

- Stall is when  $\omega_m = 0$

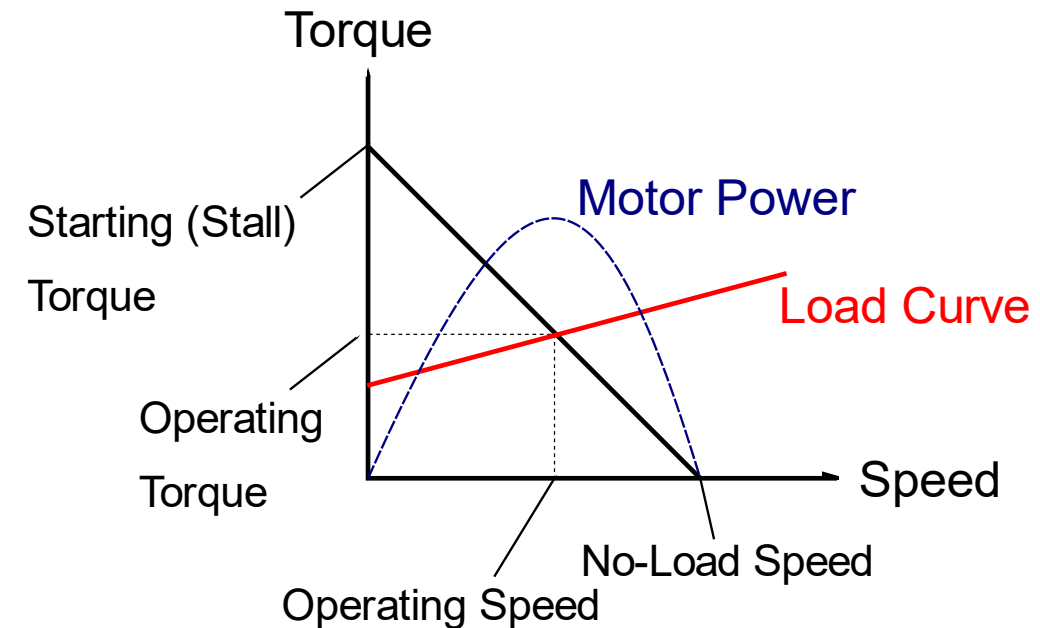
- $$T_m(t) = \frac{K_t}{R_a} e_a(t) : \text{Stall Torque}$$

- No Load Speed is when  $T_m(t) = 0$ ,

- $$\omega_{no\ load} = \frac{e_a}{K_B} : \text{No Load Speed}$$

- The constants can then be computed from the dyno generated curves.

**Ideal** Steady-State Torque-Speed Curve for a Brushed DC Motor



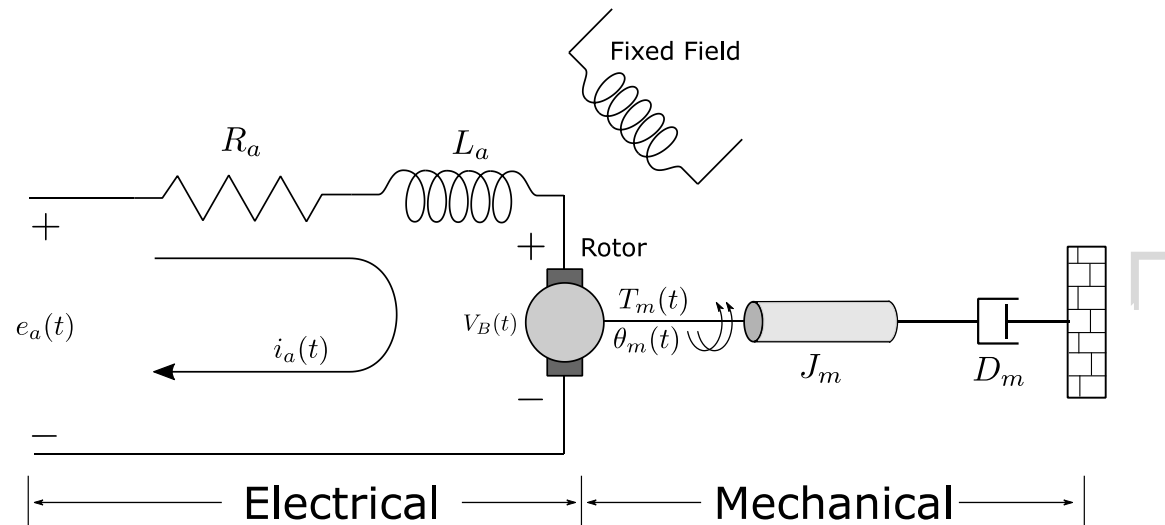
# Summary

- DC Motor Model, Combining:

- Electrical Part Gives:  $R_a I_a(s) + L_a s I_a(s) + V_B(s) = E_a(s)$
- Mechanical Part Gives:  $T_m(s) = (J_m s^2 + D_m s) \Theta_m(s)$
- Additional relationship 1:  $V_B(s) = K_B s \Theta_m(s)$
- Additional relationship 2:  $T_m(s) = K_t I_a(s)$

- We get:

$$\frac{\Theta_m(s)}{E_m(s)} = \frac{K_t / (R_a J_m)}{s \left[ s + \frac{1}{J_m} \left( D_m + \frac{K_t K_B}{R_a} \right) \right]}$$



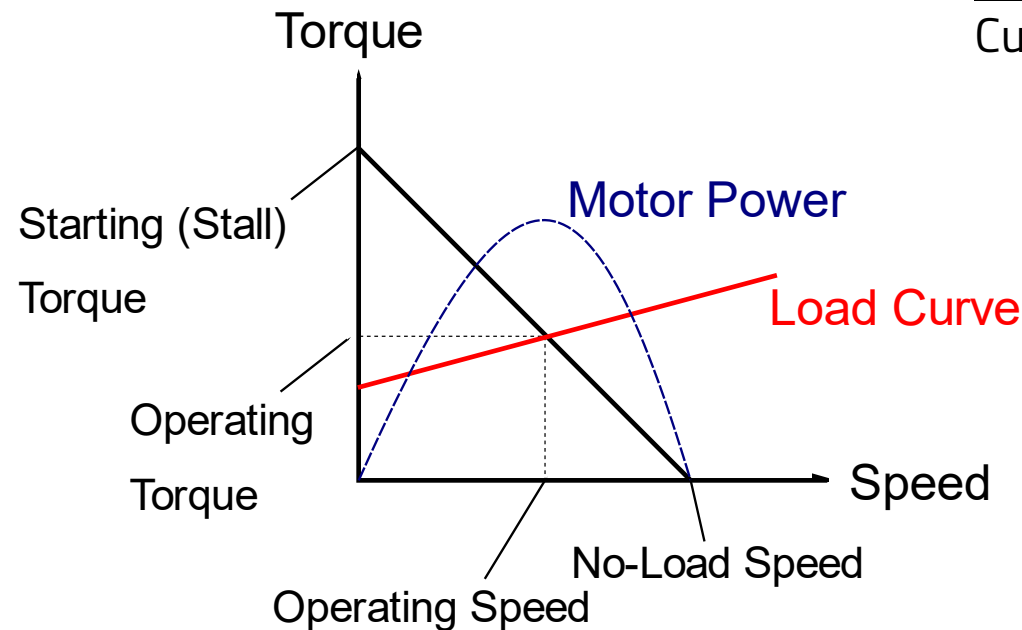
# Summary

## • Torque-Speed Curve

$$T_m(t) = \underbrace{\frac{K_t}{R_a} e_a(t)}_{\text{Stall Torque}} \underbrace{- \frac{K_B K_t}{R_a}}_{\text{Slope of Torque Speed Curve}}$$

$$\omega_m(t) = \tau_{stall} - \text{Slope}_{\tau-\omega} \omega(t)$$

**Ideal** Steady-State Torque-Speed Curve for a Brushed DC Motor





A 1/4 Hp DC motor is used to lift a 10 kg load via a pulley as shown. From the datasheet, the no-load motor speed is 300 rpm and starting torque is 23.8 N-m. Frictional resistance in pulley is 2 N-m (constant). Neglect inertia of rotor, pulley, and cable. Determine:

- Initial acceleration of load
- Steady-state speed of load
- Output horsepower of motor

### a. Initial Acceleration:

Total Torque at Startup  $\tau_{Total} = \tau_{starting} - \tau_{friction} - \tau_{gravity}$

$$\tau_{total} = 23.8 - 2 - 10 \cdot 9.81 \cdot 0.15 \text{ N} - \text{m} = 7.1 \text{ N} - \text{m}$$

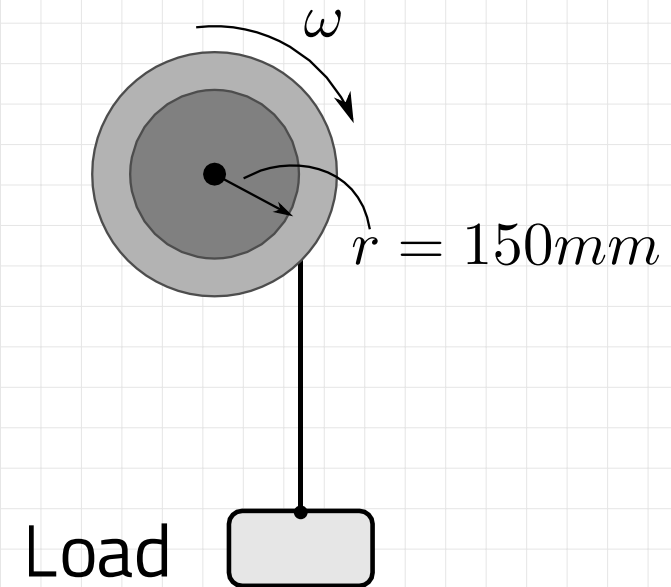
Acceleration of load due to this torque

$$F_{total} = \frac{\tau_{total}}{r} = \frac{7.1 \text{ N} - \text{m}}{0.15 \text{ m}} = 47.3 \text{ N} \rightarrow a = \frac{F}{m} = \frac{47.3 \text{ N}}{10 \text{ kg}} = 4.731 \text{ m/s}^2$$

### b. Steady-State Speed

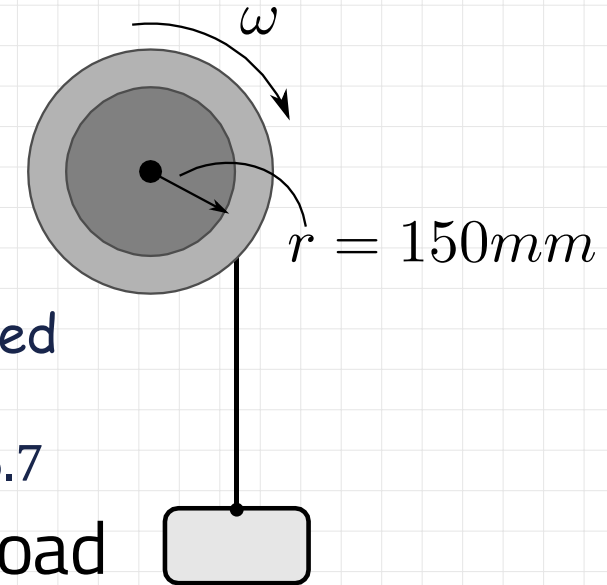
At steady state, the load is not accelerating, and the net torque exerted is

$$\tau_{ss} = \tau_{frictional} + \tau_{gravity} = 2 + 10 \cdot 9.81 \cdot 0.15 = 16.7 \text{ N} - \text{m}$$



A 1/4 Hp DC motor is used to lift a 10 kg load via a pulley as shown. From the datasheet, the no-load motor speed is 300 rpm and starting torque is 23.8 N-m. Frictional resistance in pulley is 2 N-m (constant). Neglect inertia of rotor, pulley, and cable. Determine:

- Initial acceleration of load
- Steady-state speed of load
- Output horsepower of motor



Using the torque-speed equation, we can determine the steady-state speed

$$\tau_{motor} = \tau_{stall} - \text{Slope}_{\tau-\omega} \omega(t) = 23.8 - \underbrace{(23.8/300)}_{\text{Slope of torque-speed curve}} \omega = \tau_{ss} = 16.7$$

$$\omega = 89.5 \text{ RPM} \rightarrow v = \omega r = 89.5 \text{ RPM} \cdot \frac{2\pi}{60} \cdot 0.15 \text{ m} = 1.41 \text{ m/s}$$

### c. Output Horsepower of Motor

$$P = \tau \omega = 16.7 \text{ N-m} \cdot 89.5 \text{ RPM} \cdot \frac{2\pi}{60} \frac{1 \text{ Hp}}{746 \text{ Watts}} = 0.21 \text{ Hp} < P_{rated} = 0.25 \text{ Hp}$$

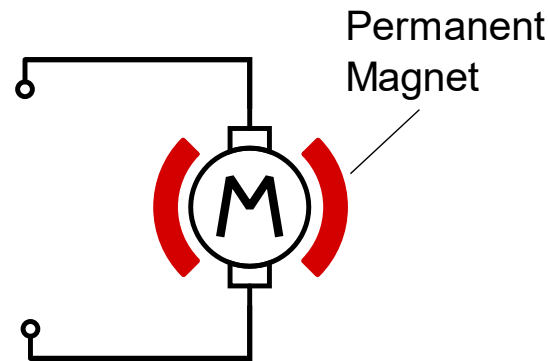


# Conventional DC Motor Types

- There are 4 primary types of DC motors

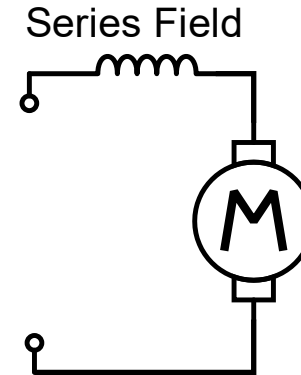
## PM DC Motor

- PM supplies flux field
- Good starting torque
- Can demagnetize permanent magnets if too much current supplied



## Series-Wound DC Motor

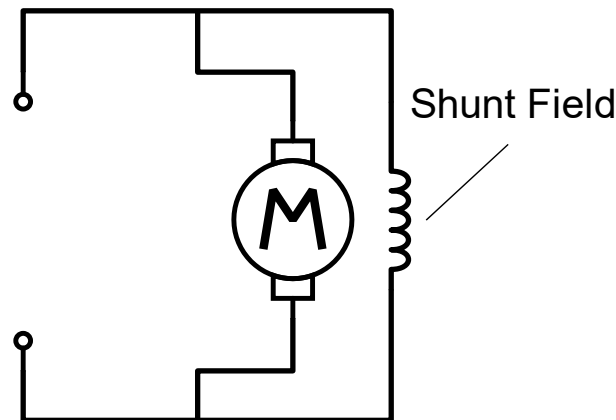
- Offers large starting torque
- Will fail if run with load disconnected



- Good starting torque and speed regulation

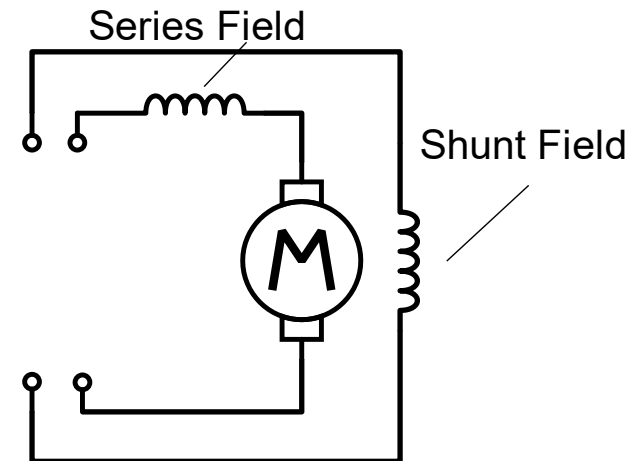
## Shunt-Wound DC Motor

- Offers nearly constant speed under varying loads (good speed regulation)
- Found in machine tools



## Compound-Wound DC Motor

- Good balance of series- and shunt-wound characteristics

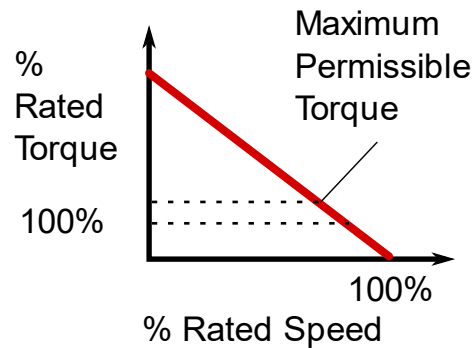


# Conventional DC Motor Torque vs Speed Curves

- DC motors provide a varying amount of torque depending on operating speed
  - Generally torque decreases as motor runs faster

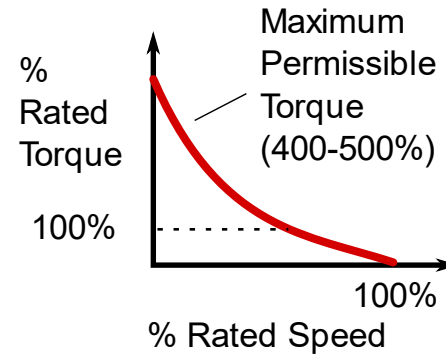
## PM DC Motor

- Torque vs speed curve is linear for a DC motor



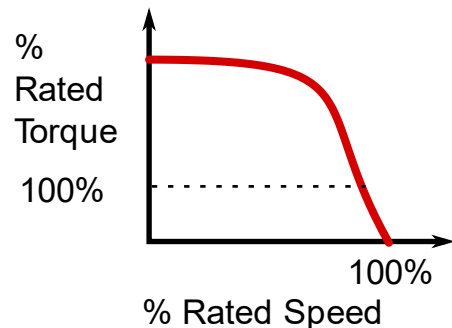
## Series-Wound DC Motor

- Large starting torque
- But motor speed grows uncontrollably if zero torque is applied



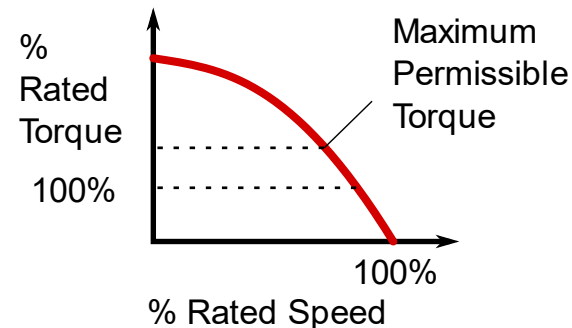
## Shunt-Wound DC Motor

- Relatively flat torque-speed characteristics

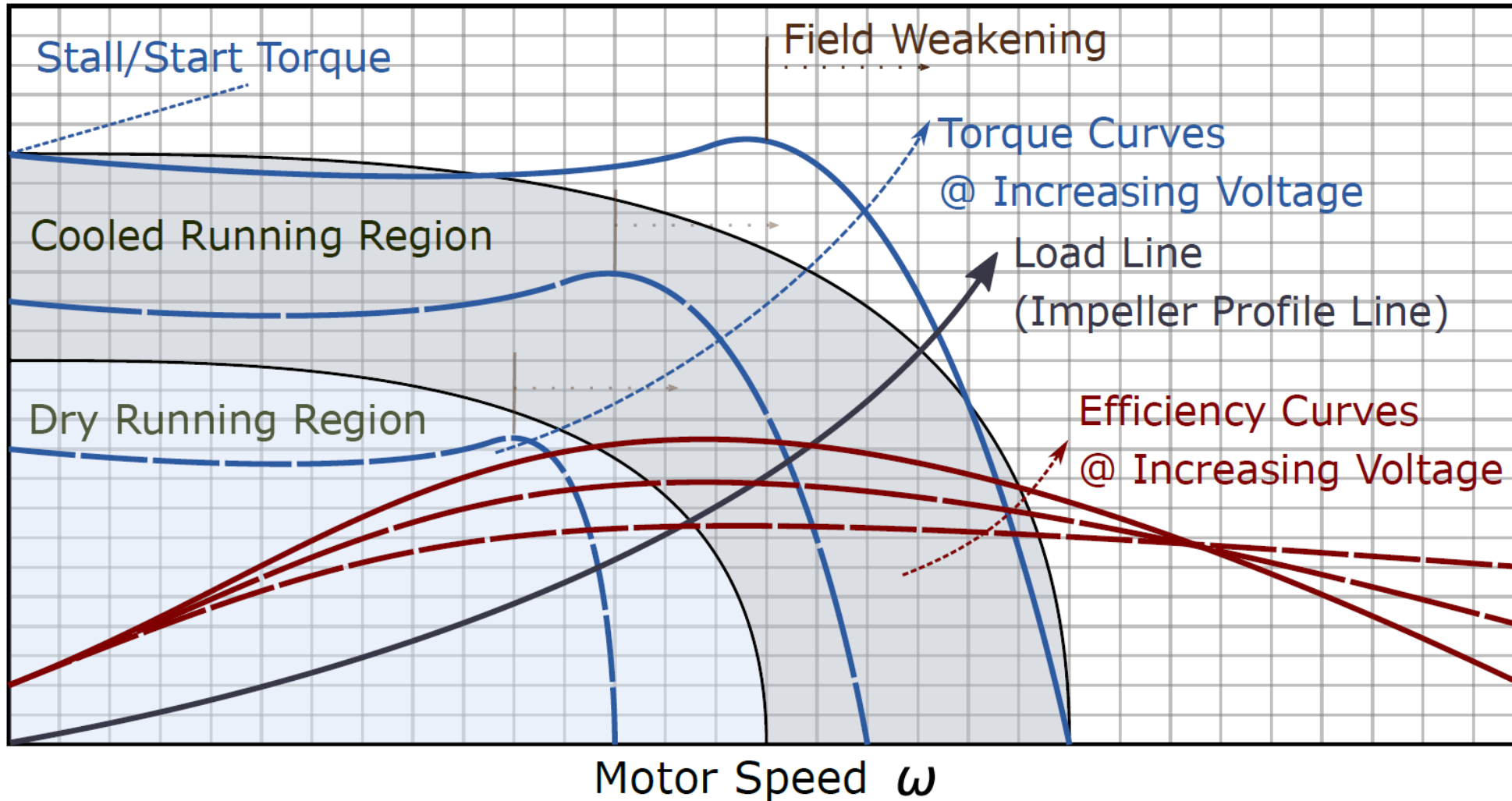


## Compound-Wound DC Motor

- Good starting torque and speed regulation
- Good balance of series- and shunt-wound characteristics



# Motor Curve – Conceptual



*Mechatronic design of autonomous underwater vehicles for confined spaces, AlSaibie, PhD Thesis, 2018*



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# Brushless PM DC Motors

- In brushed DC motors, brushes create mechanical point of contact between stator and rotor
  - Necessary in order to power wire coils on rotor
  - Generate heat and acoustic noise, must be replaced periodically
- Brushless DC motors do not use brushes
  - Only points of contact between rotor and stator are bearings
  - No direct wiring to rotor



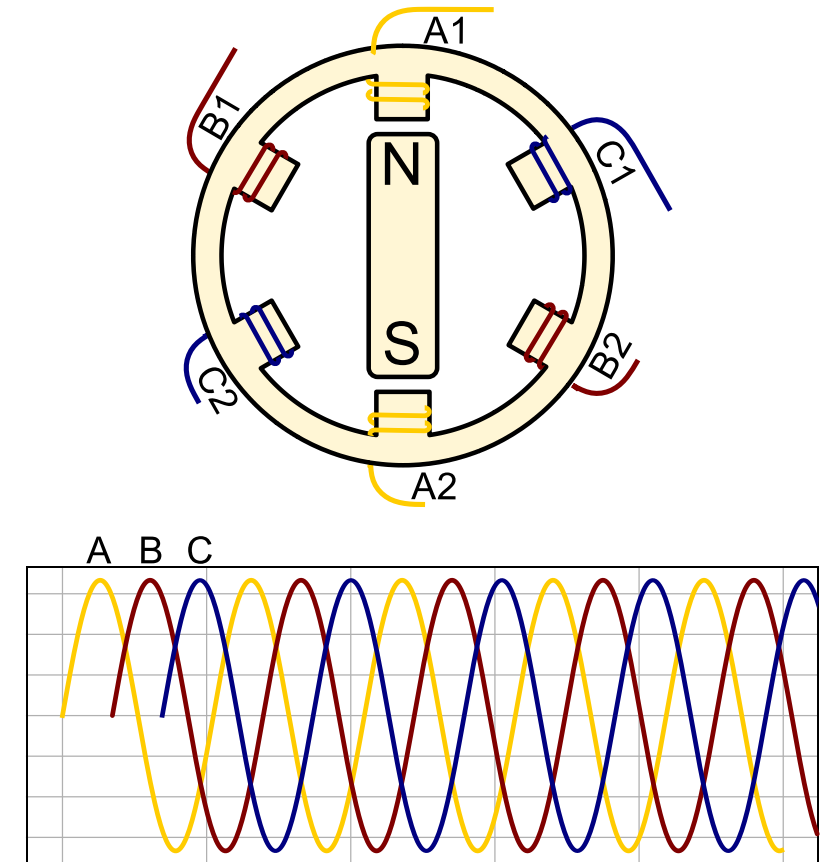
# Brushless Motor Names

- These are essentially the same type of motor:
  - Synchronous Machine
  - Permanent Magnet Synchronous Machine
  - Brushless Permanent Magnet Synchronous Motor
  - Brushless A.C. Motor
  - Brushless D.C. Motor
  - Permanent Magnet Servo Motor
- The different names serve to confuse us.
  - They may vary a bit, but principle of operation is the same.
- Just use the term: Brushless Motor (most common in mechatronics/robotics)



# Brushless Motor Operation

- In brushless DC motor, rotor is made of permanent magnet and stator is made of coils
  - This is opposite of brushed motors
- Concept of operation:
  - Position of magnet is detected
    - Through hall-effect sensor
    - Or by measuring current (sensor-less operation)
  - Coil pairs are activated sequentially so that magnetic field is always perpendicular (as much as possible) to rotor magnet
  - Causes rotor to spin
  - Thus commutation is done electrically and not mechanically
    - In a conventional DC motor, the current switches direction when the shaft commutator rotates flips current direction





# Brushless Motor

- Is it AC or DC?
  - It's a multi-phase (3-phase most commonly), that operates via principle of switching current direction (ac).
  - Why do they call it a Brushless DC motor sometimes?
    - Because the "controller" can be supplied a DC current, and the "controller" produces the ac like currents to the motor.
    - So to be specific the "controller" is dc. The motor itself is not.



# Brushless DC Motor Advantages

- BLDC Rotor is lighter than on brushed motors
  - (+) BLDC's can operate at much higher speeds than DC motors
- BLCD does not use mechanical brushes for commutation
  - (+) BLDC's are more reliable since they do not generate much heat due to friction
  - (+) BLDC's are quieter
  - (+) BLDC's are more efficient since there are less frictional losses
  - (-) BLDC's require more complex circuitry to operate. They are also more expensive.

