Kuwait University
College of Engineering and Petroleum

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ME417 CONTROL OF MECHANICAL SYSTEMS

Part I: Introduction to Feedback Control

Lecture 4: Mechanical Systems Transfer Functions

Summer 2020

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- Objectives:
  - *Review Dynamic Modeling of Translational Mechanical Systems*
  - *Review Dynamic Modeling of Rotational Mechanical Systems*
- Reading:
  - *Nise: 2.5.-2.7*
- Practice Problems Included

- The general form of the equation of motion for a mechanical system is:

$$I(t,\ddot{q}) + D(t,q,\dot{q}) + K(q,t) = f(t,q)$$

Where I, D, K denote the inertial, damping and spring terms respectively, q is the general position coordinate, and f denotes the input force.

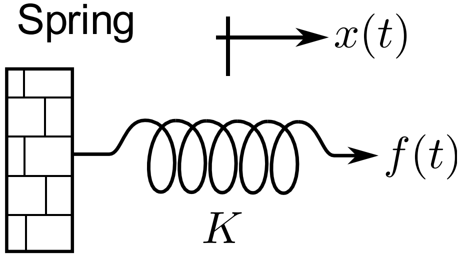
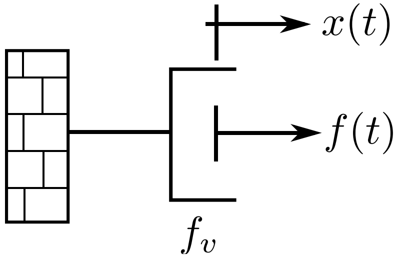
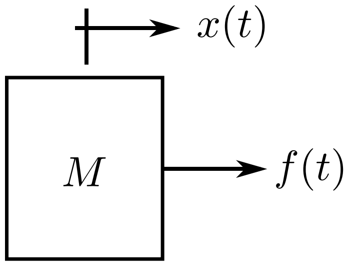
- In this course we treat linear, time-invariant systems
  - I, D, K are constants; they are
    - Not a function of time: time-invariant
    - Not a function of q or any other variable: Linear
- The general form of the equation of motion for a linear, time-invariant mechanical system is thus:

$$I\frac{d^2q(t)}{dt} + D\frac{dq(t)}{dt} + Kq(t) = f(t)$$

For translational systems we use $x(t)$ instead of $q(t)$

For rotational system we use $\theta(t)$ instead of $q(t)$

Table 2.4

Component	Force-Velocity	Force-Displacement	Impedance
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous Damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

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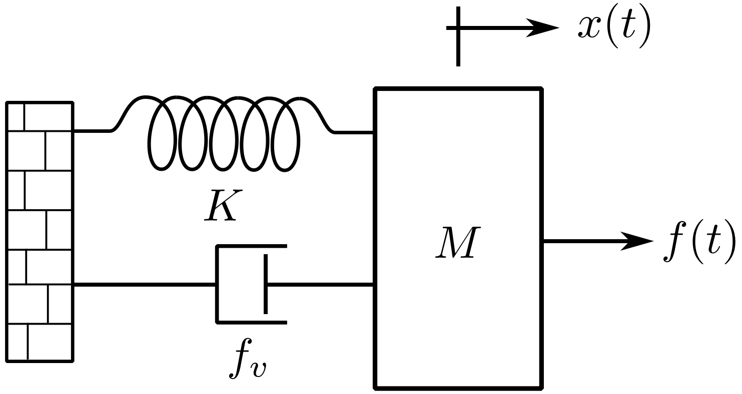
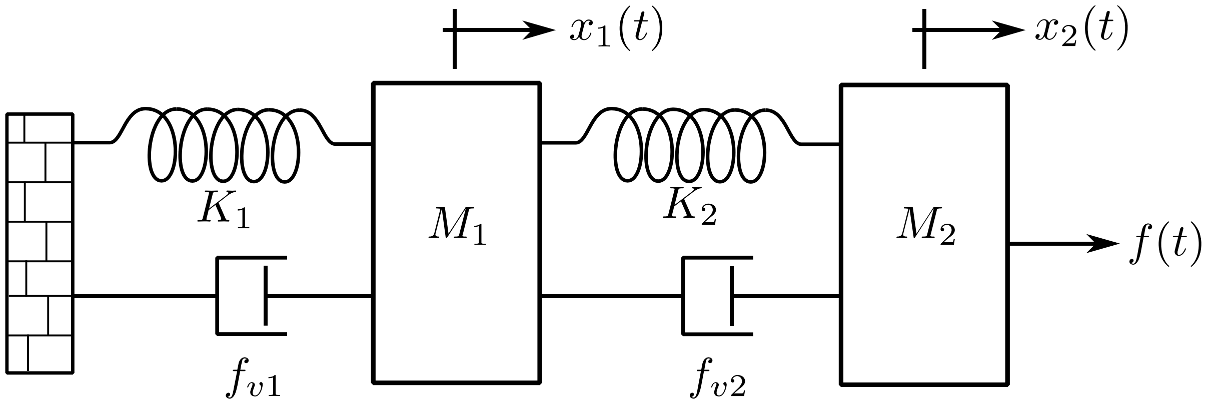
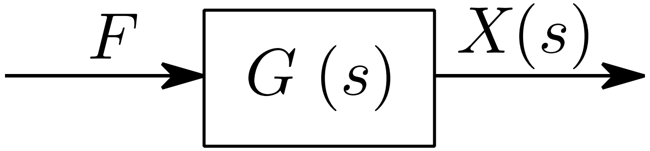
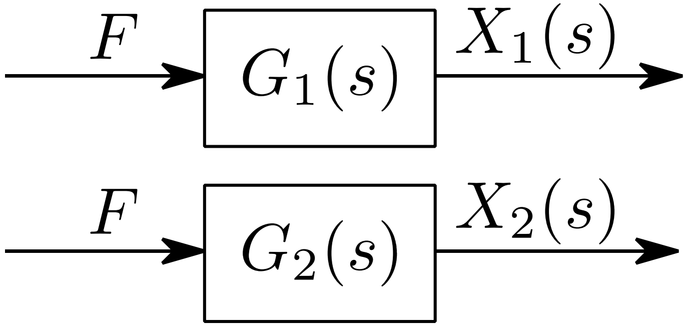
- A system with one degree of freedom will give one equation of motion and thus one transfer function for one input.
- A system with two degrees of freedom will give two equations of motion, and thus two transfer functions for one input, etc.

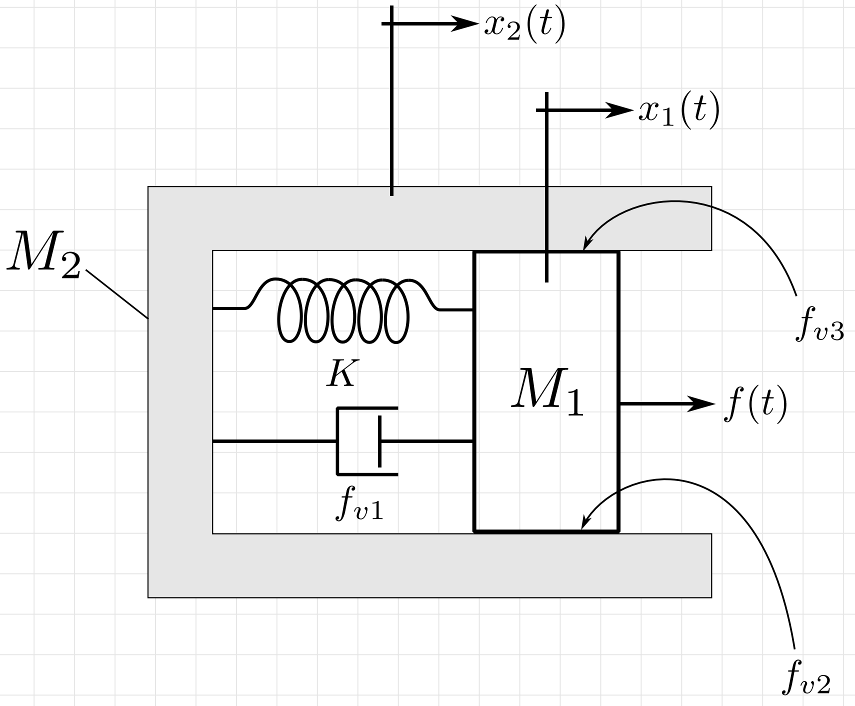








Write the Laplace Transform of the equations of motion for the system shown



- Impedance of a mechanical system is defined as $Z_m(s) = \frac{F(s)}{X(s)}$
- We can derive the EOM by visual inspection by noting that

$$[\text{Sum of Impedances}]\, X(s) = [\text{Sum of Applied Forces}]$$

- For a two-degree of freedom system

	[Sum of Impedances connected to the motion at x_1]	$X_1(s)$ **-**	[Sum of Impedances between x_1 & x_2]	$X_2(s)$ **=**	[Sum of Applied Forces at x_1]
-	[Sum of Impedances between x_1 & x_2]	$X_1(s)$ **+**	[Sum of Impedances connected to the motion at x_2]	$X_2(s)$ **=**	[Sum of Applied Forces at x_2]

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- The solution for $Ax = y$, where A is an invertible matrix is $x = A^{-1}y$
- When we have n **linear** equations with n unknowns, we can use Cramer's rule.
- Given two linear equations

$$a_1x_1 + b_1x_2 = y_1$$

$$a_2x_1 + b_2x_2 = y_2$$

- We can find $x_1 = f(y), x_2 = f(y)$ using Cramer's rule

$$x_1 = \frac{b_2y_1 - b_1y_2}{\Delta}, x_2 = \frac{a_1y_2 - a_2y_1}{\Delta}$$

$$\Delta = \det(A) = a_1b_2 - a_2b_1, A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

- Cramer's rule is useful in decoupling the variables when deriving the Transfer Function from the Laplace Transform of 2nd or higher order linear systems.

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Example 2

Find the transfer function $G_2(s) = \frac{X_2(s)}{F(s)}$, for the system shown on the figure, using the impedance method.

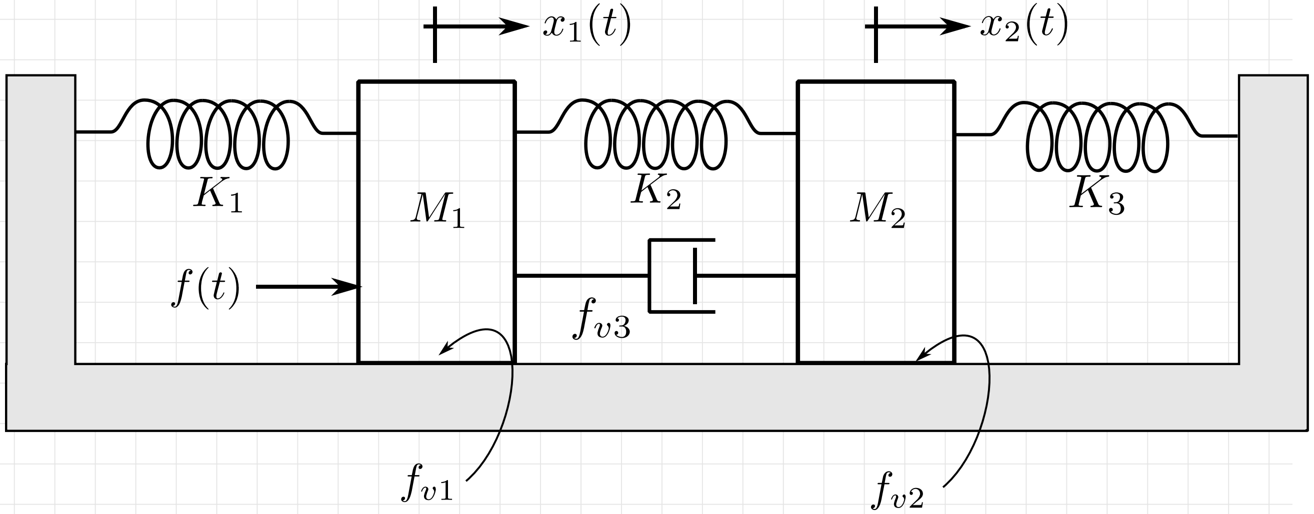

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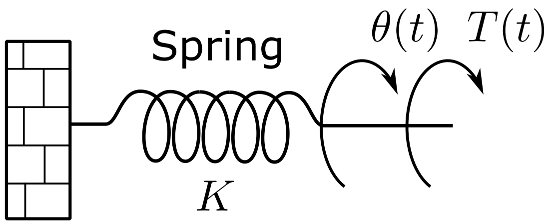
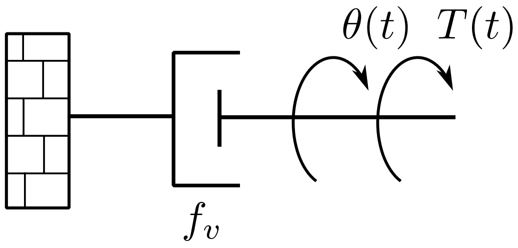
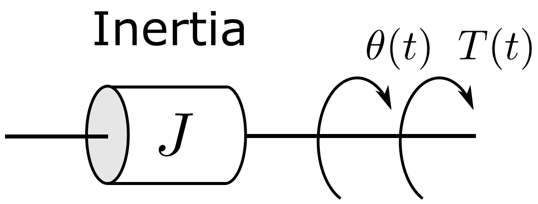
Table 2.5

Component	Torque – Angular Velocity	Torque-Angular Displacement	Impedance
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
Viscous Damper 	$T(t) = f_v\omega(t)$	$T(t) = f_v\frac{d\theta(t)}{dt}$	$f_v s$
	$T(t) = J\frac{d\omega(t)}{dt}$	$T(t) = J\frac{d^2\theta(t)}{dt^2}$	Js^2

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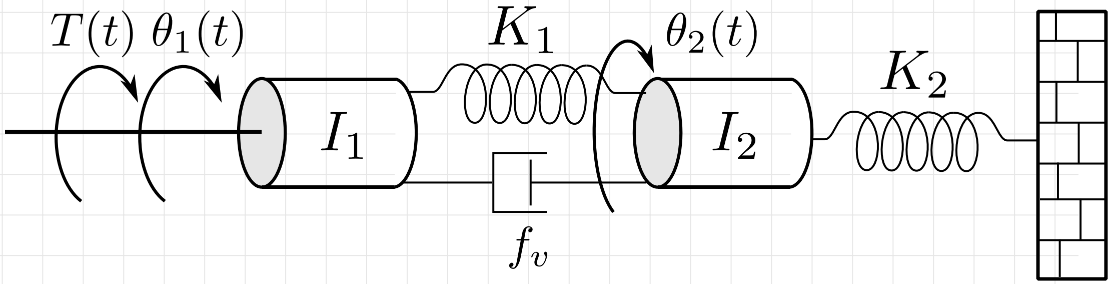
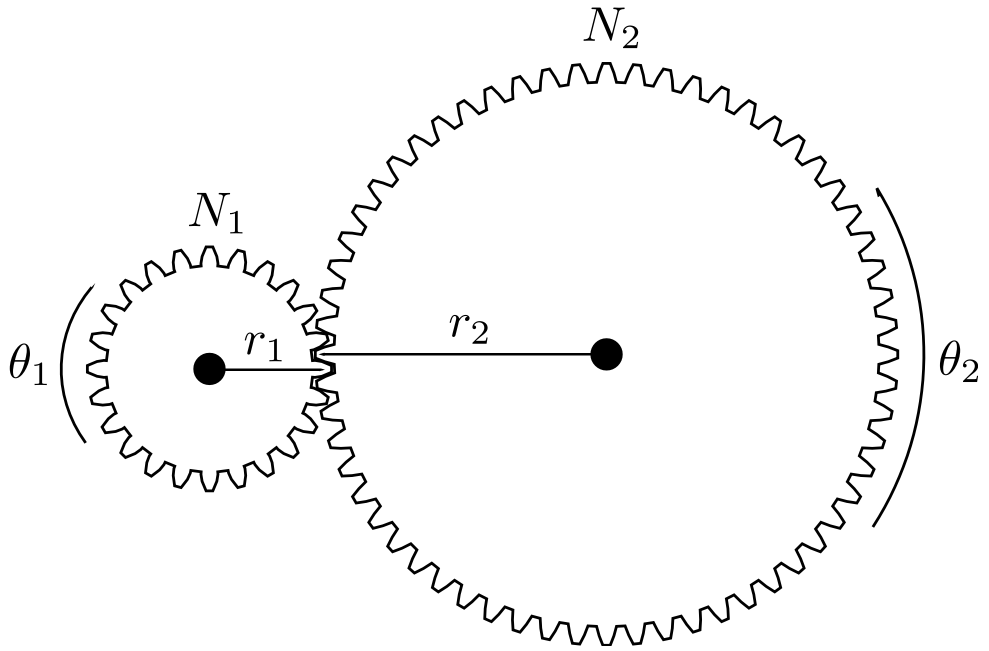
Example 3

Draw the free body diagram for the system shown, then find the transfer function $G_1(s) = \frac{\theta_1(s)}{T(s)}$, using the impedance method.



- In this course we assume lossless gears
  - No backlash in gears
  - Linear interaction between the gears:

$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = \frac{T_1}{T_2}$$



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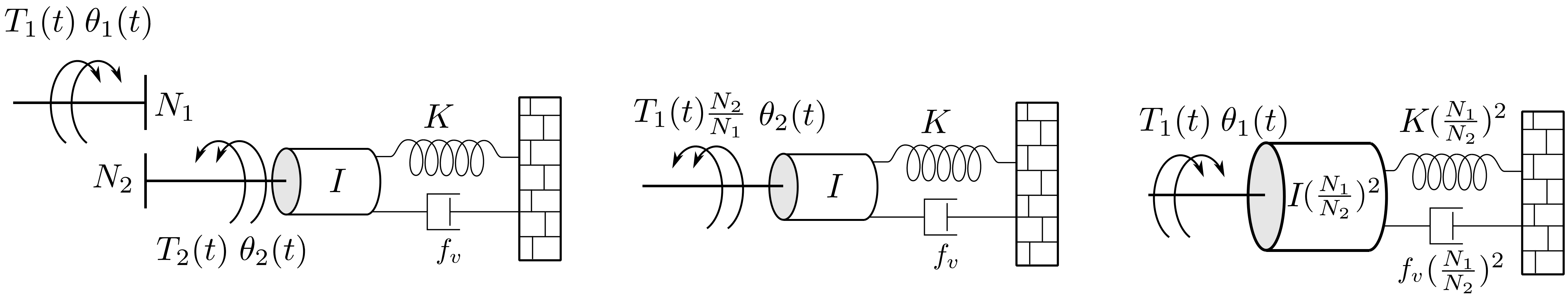

$$(Is^2 + f_v s + K)\theta_2(s) = T_1(s)\frac{N_2}{N_1} \Rightarrow (Is^2 + f_v s + K)\frac{N_1}{N_2}\theta_1(s) = T_1(s)\frac{N_2}{N_1}$$

$$\Rightarrow \left(I\left(\frac{N_1}{N_2}\right)^2 s^2 + f_v\left(\frac{N_1}{N_2}\right)^2 s + K\left(\frac{N_1}{N_2}\right)^2\right)\theta_1(s) = T_1(s)$$

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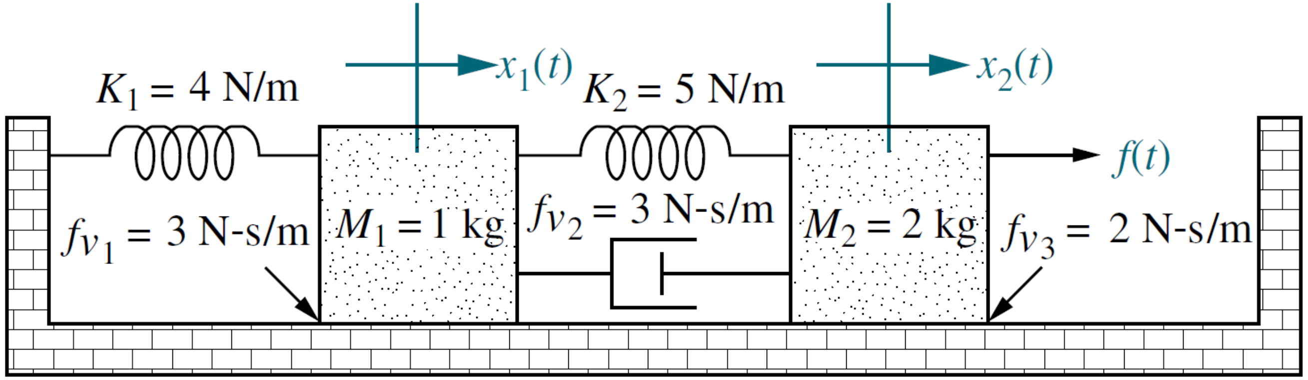
Practice Problem 1

Find the transfer function $G_1(s) = \frac{X_1(s)}{F(s)}$, for the system shown. Nise: Problem 2-26



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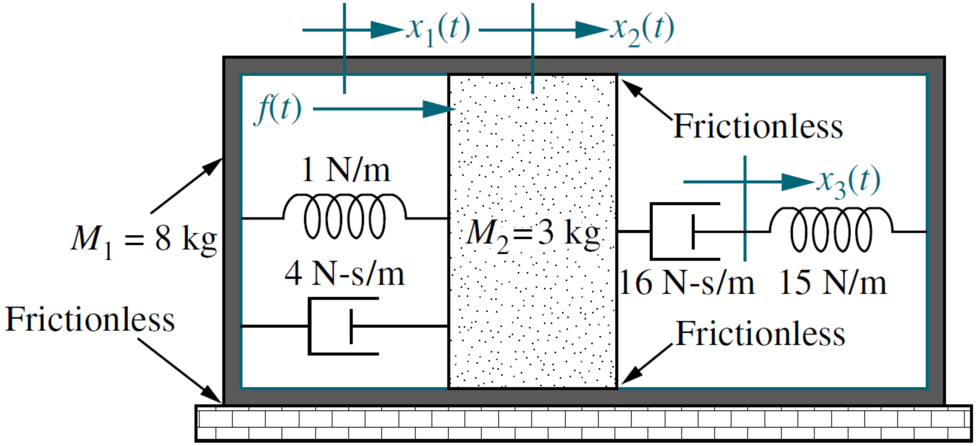
Practice Problem 2

Find the transfer function $G_3(s) = \frac{X_3(s)}{F(s)}$, for the system shown. Nise: Problem 2-28b



(b)

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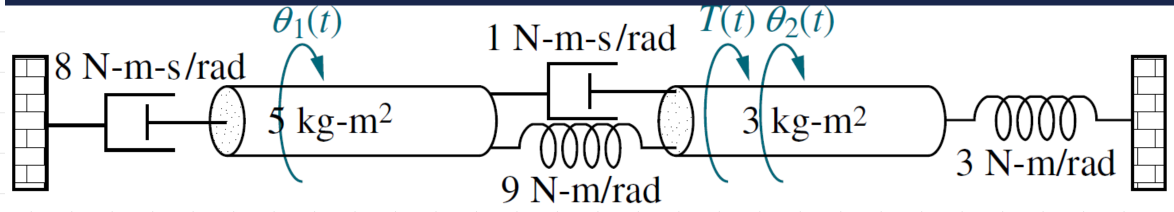
Practice Problem 3

Find the transfer function $G_1(s) = \frac{\theta_1(s)}{T(s)}$, for the system shown. Nise: Problem 12-31a



Find the transfer function, $G_2(s) = \frac{\theta_2(s)}{T(s)}$, for the system shown.
Nise: Problem 2-35.