Kuwait University
College of Engineering and Petroleum

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ME417 CONTROL OF MECHANICAL SYSTEMS

Part I: Introduction to Feedback Control

Lecture 5: Electromechanical Systems Transfer Functions

Summer 2020

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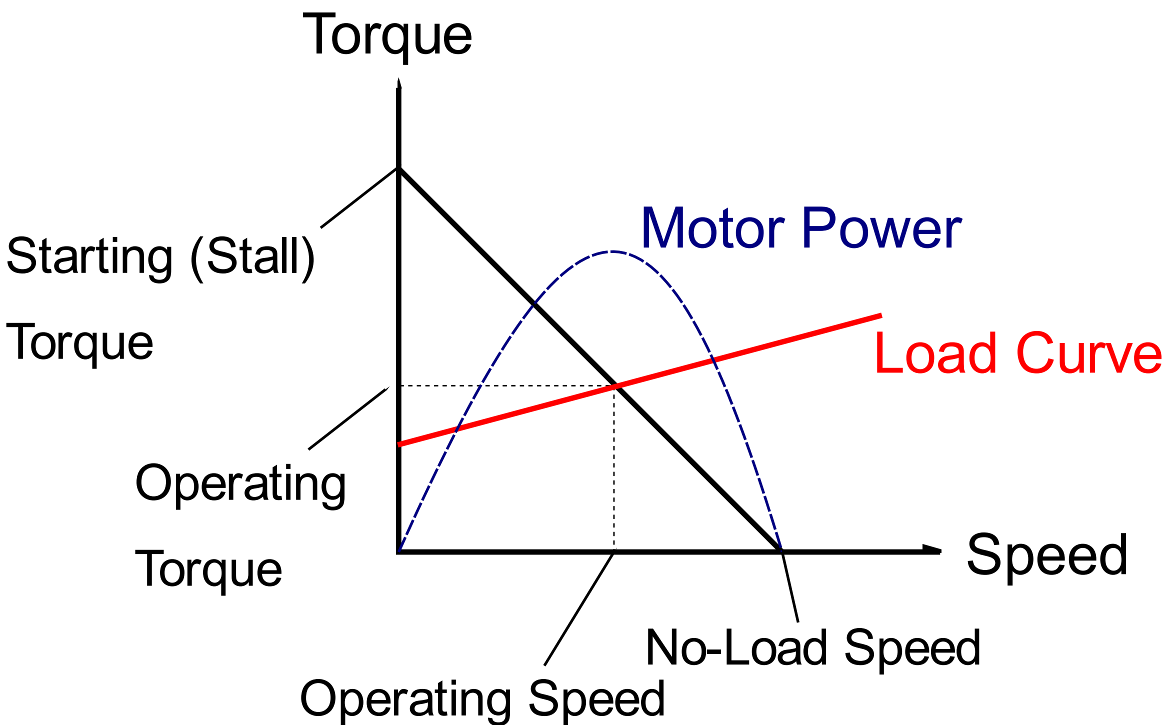
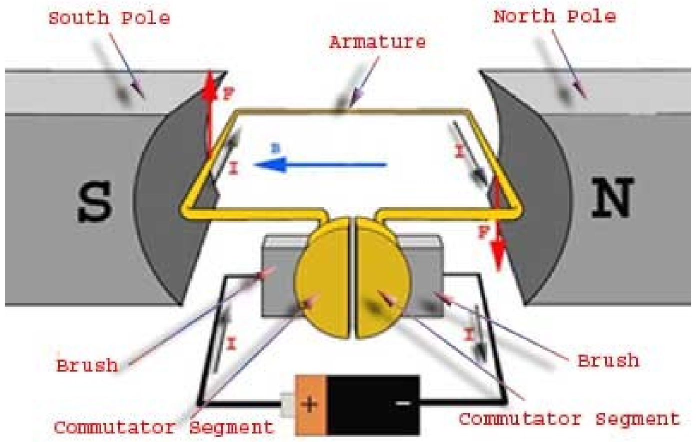
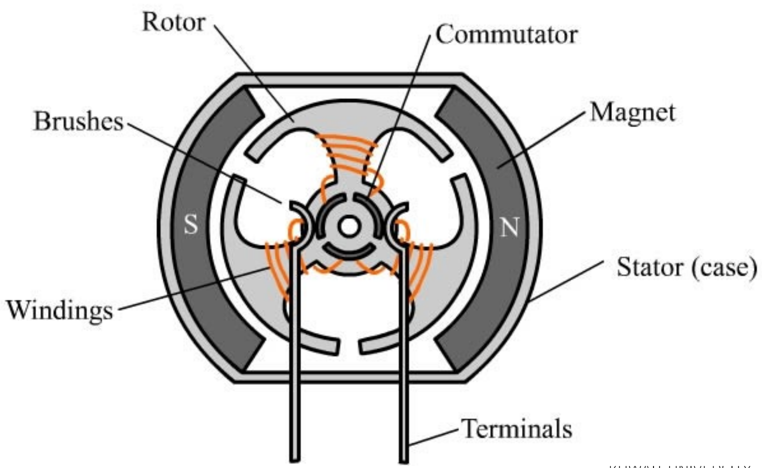
- Objectives:
  - *Review Dynamic Modeling of Electromechanical Systems*
- Reading:
  - *Nise: 2.8*
- Practice Problems Included

- Electromechanical systems: Part Electric/Electrical Part Mechanical
- A classic example is the Brushed DC Motor



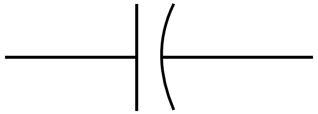
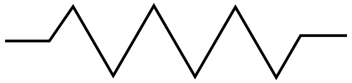
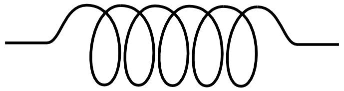


Typical Brushed Motor in Cross-section



Electrical Systems Components

Table 2.3

Component	Voltage-Current	Voltage-Charge	Impedance
Capacitor 	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$
Resistor 	$v(t) = Ri(t)$	$v(t) = R\frac{dq(t)}{dt}$	R
Inductor 	$v(t) = L\frac{di(t)}{dt}$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls

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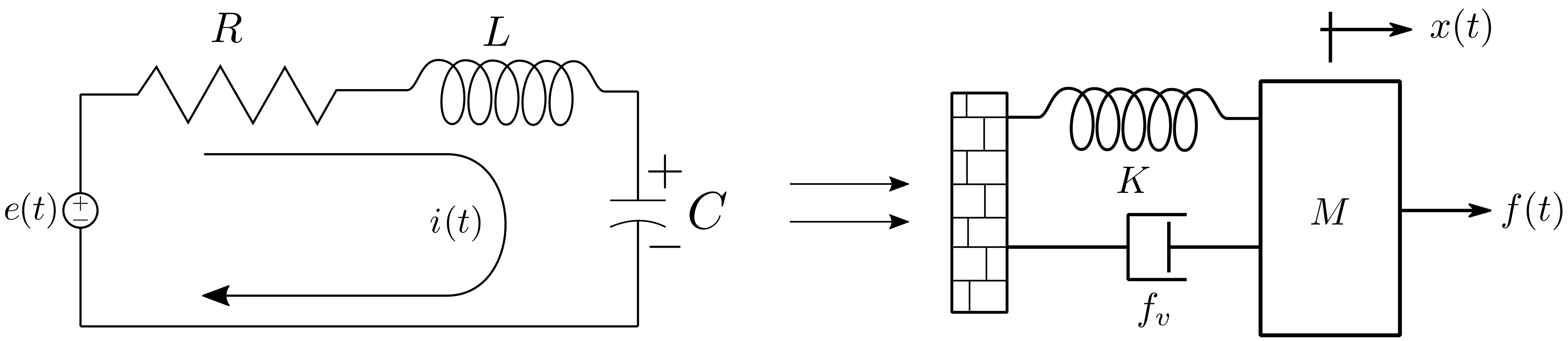
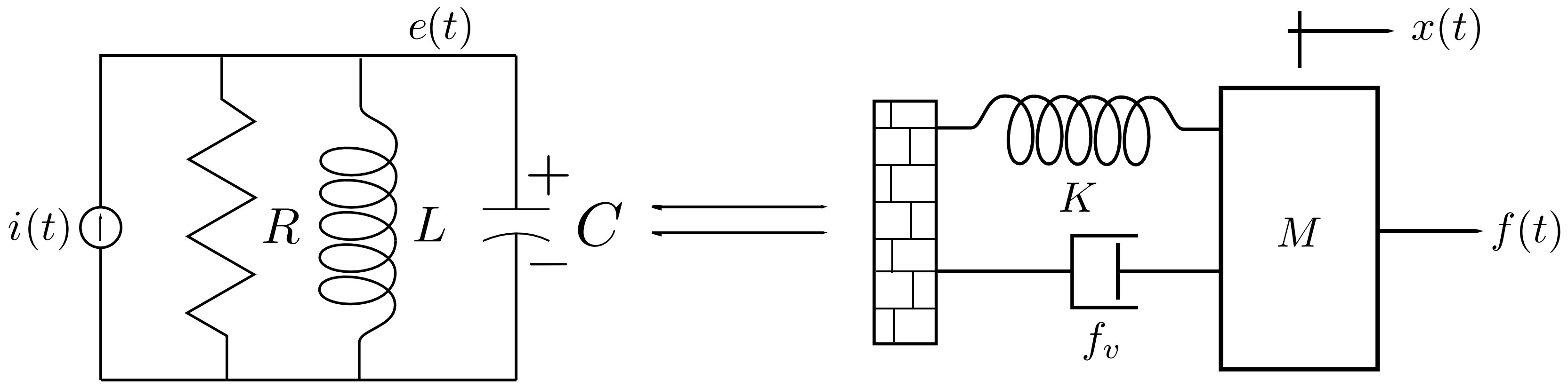
- Mass M ⇔Inductor L
- Viscous Damper f_v ⇔Resistor R
- Spring K ⇔Capacitor C
- Applied Force $f(t)$ ⇔Voltage Source $e(t)$
- Velocity $v(t)$ ⇔Current $i(t)$

$$\left(Ls + R + \frac{1}{Cs}\right)I(s) = E(s) \Leftrightarrow (Ms^2 + f_v s + K)X(s) = F(s)$$

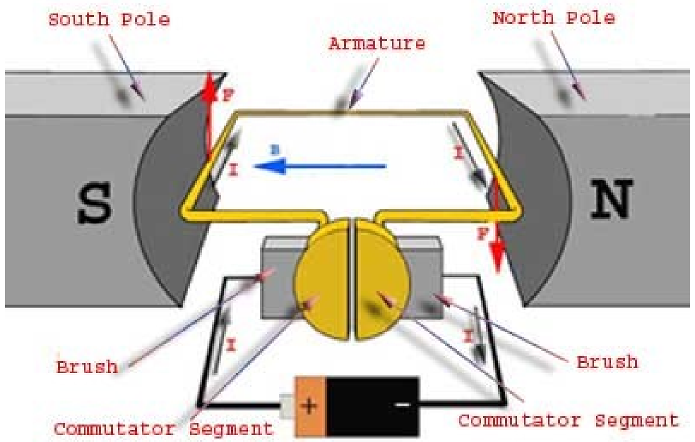


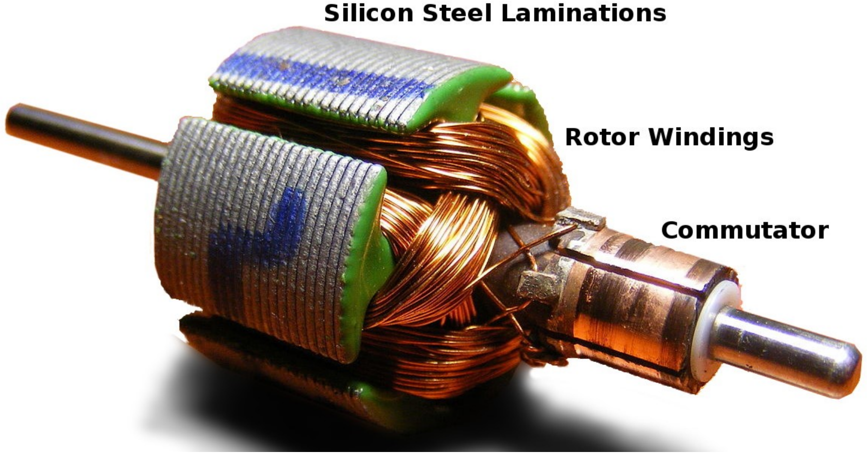
- Mass M ⇔ Capacitor C
- Viscous Damper f_v ⇔Resistor R
- Spring K ⇔Inductor L
- Applied Force $f(t)$ ⇔Current Source $i(t)$
- Velocity $v(t)$ ⇔Node Voltage $v(t)$

$$\left(Cs + \frac{1}{R} + \frac{1}{Ls}\right)E(s) = I(s) \Leftrightarrow (Ms^2 + f_v s + K)X(s) = F(s)$$

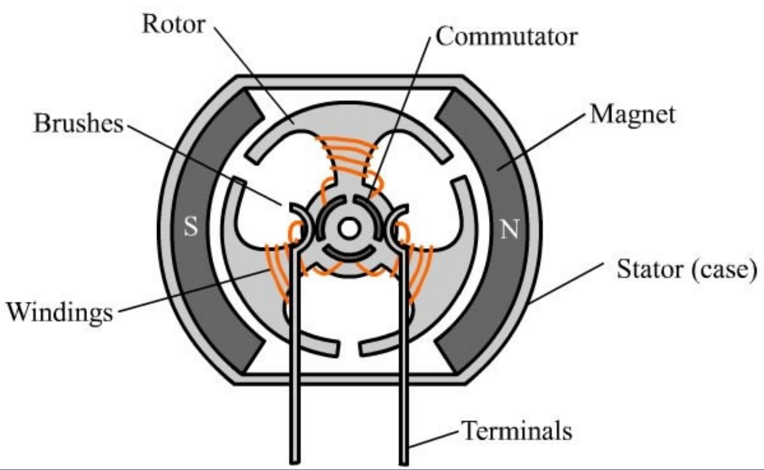


- Converts Electrical Energy to Mechanical Energy
- DC Motor Consists of
  - Coils/Windings called Armature
  - Permanent magnets called Stator
  - Conductive brushes that keep the current flowing in the armature and make it possible to reverse current direction, called commutator
    - commutes the current, as in changes its direction





Typical Brushed Motor in Cross-section

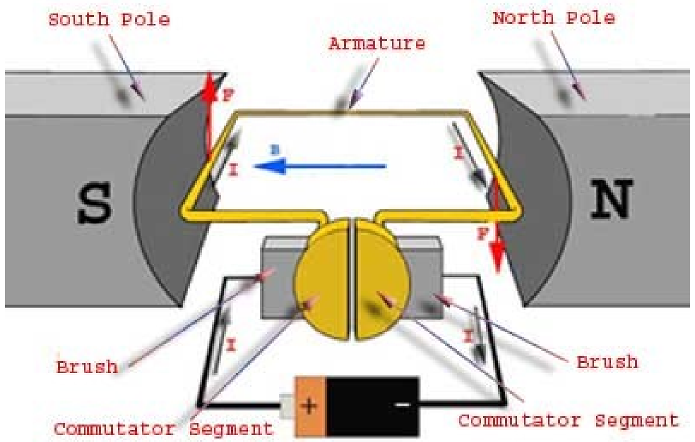


- By applying voltage across the coil, current $i(t)$ is generated, flowing through a moving coil. This generates an electromagnetic field.

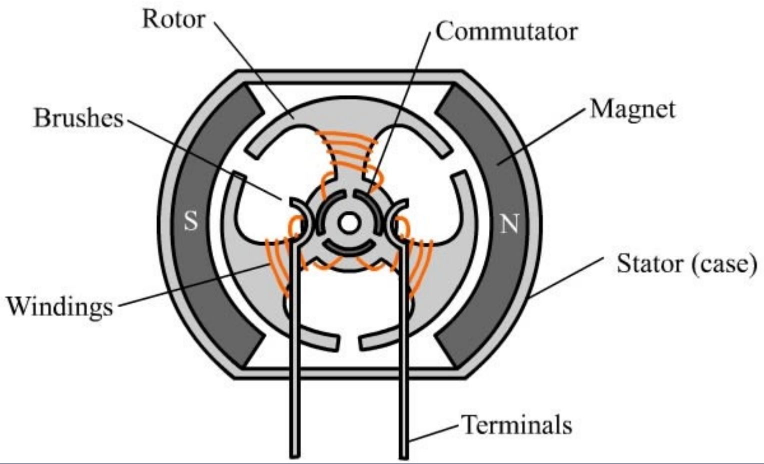
- Moment-Arm between the permanent magnet and the generated electromagnetic field generate torque.
  - $F = Bli(t)$, B is the magnetic field strength, l is the conductor length

- Commutator periodically flips the direction of current to continue this effect (*"Opposites Attract"*)

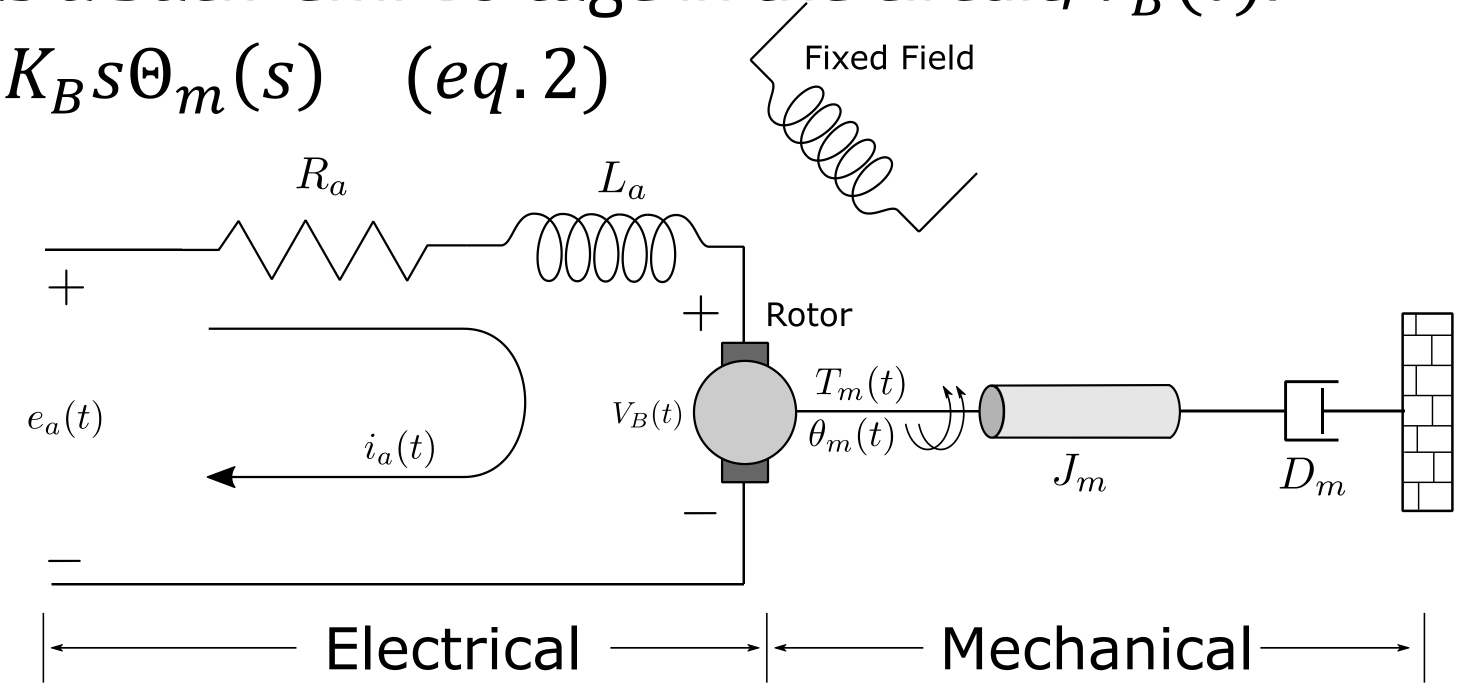
- The magnetic field generates a different voltage across the moving coil called back electro-motive force, back-emf.
  - $V_B = Bl\omega$, where ω is the angular velocity of the rotating coil.



Typical Brushed Motor in Cross-section



- The DC Motor is modeled as a circuit + rotational mechanical system
- On the electrical side, we apply voltage across the coils which have resistance and inductance, modeled as R_a and L_a.
  - $R_a I_a(s) + L_a s I_a(s) + V_B(s) = E_a(s) \quad (eq. 1)$
- The motor is represented as a back-emf voltage in the circuit, $V_B(t)$.
  - $V_B = K_B \dot{\theta}_m \Leftrightarrow V_B(s) = K_B s \Theta_m(s) \quad (eq. 2)$
  - K_B: back-emf constant



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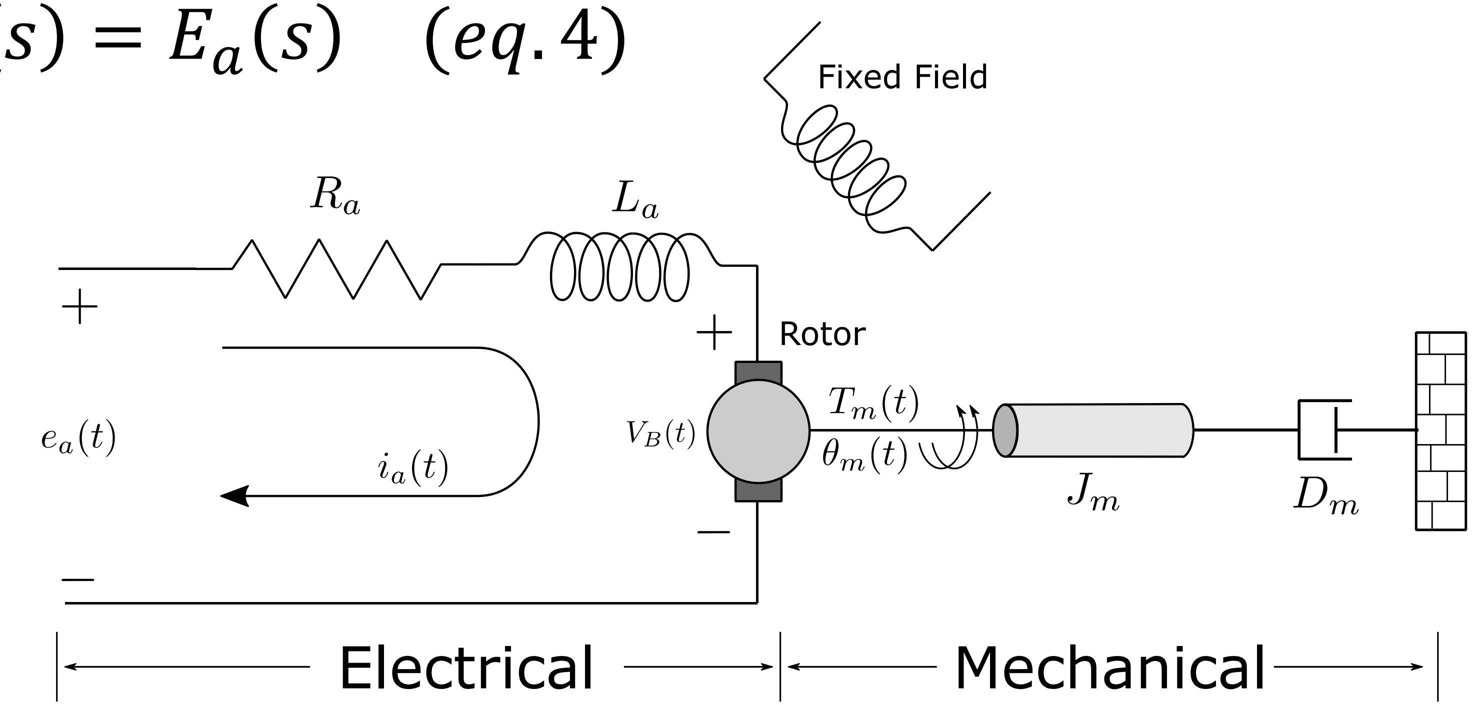
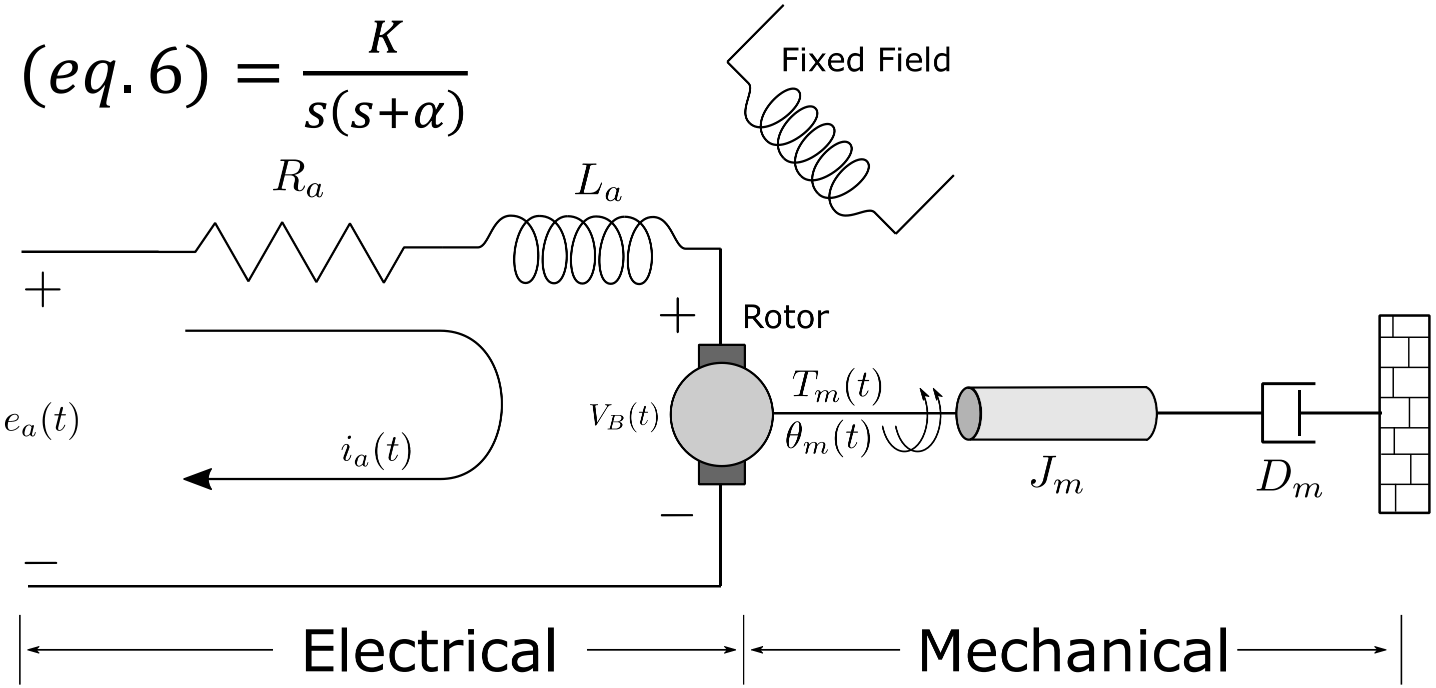
- The Fixed Field represents the permanent magnets' field.
- The torque developed by the motor is proportional to the field current.
  - $T_m(s) = K_t I_a(s) \quad (eq.3)$, K_t: motor torque constant

- Combining equations 1 to 3
  - $\frac{(R_a+L_a s)T_m(s)}{K_t} + K_B s\Theta_m(s) = E_a(s) \quad (eq.4)$



- On the mechanical side, the motor itself has an inertia J_m , that rotates with angular velocity $\dot{\theta}_m$, in addition to mechanical bearing friction (viscous damping) D_m
  - $T_m(s) = (J_m s^2 + D_m s)\Theta_m(s) \quad (eq. 5)$
- Combining equations 4 & 5, and rearranging to express $\frac{\Theta_m(s)}{E_m(s)}$, ignoring L_a as $R_a/L_a \gg 1$
  - $\frac{\Theta_m(s)}{E_m(s)} = \frac{K_t/(R_a J_m)}{s[s+\frac{1}{J_m}(D_m+\frac{K_t K_B}{R_a})]} \quad (eq. 6) = \frac{K}{s(s+\alpha)}$
- The constants K, α
  - $K = K_t/(R_a J_m)$
  - $\alpha = \frac{1}{J_m}(D_m + \frac{K_t K_B}{R_a})$



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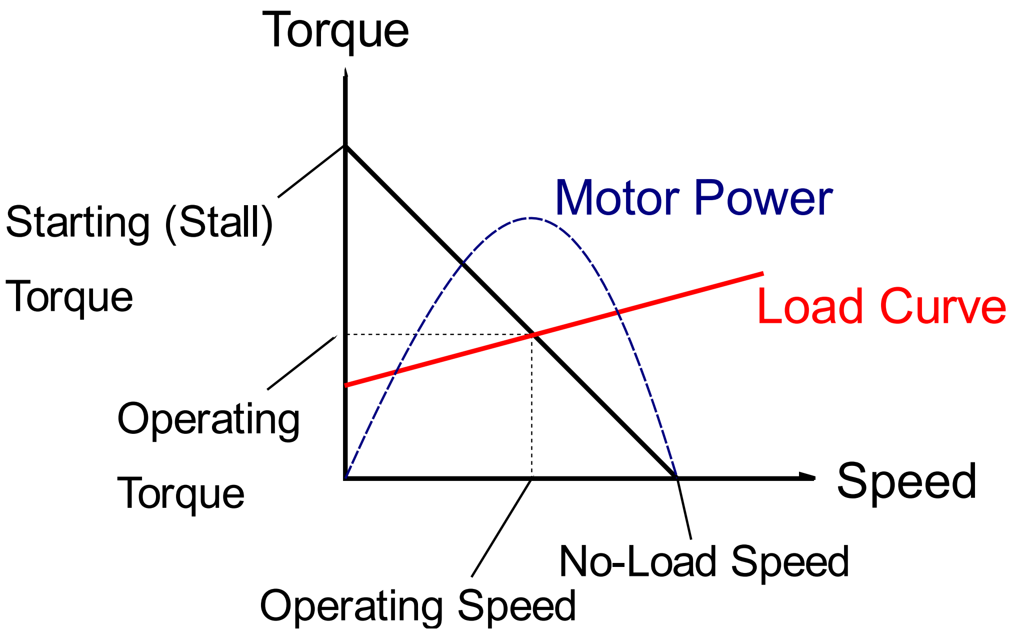
- In control system design, we are interested in modeling a real motor.
- How do we find the constants' values?
- A dynamometer can be used to generate a profile of the motor
- A dynamometer is a test bench for motors, allows for changing mechanical load, changing supplied voltage and measuring generated torque and current consumed. The generated profile is a torque-speed curve.

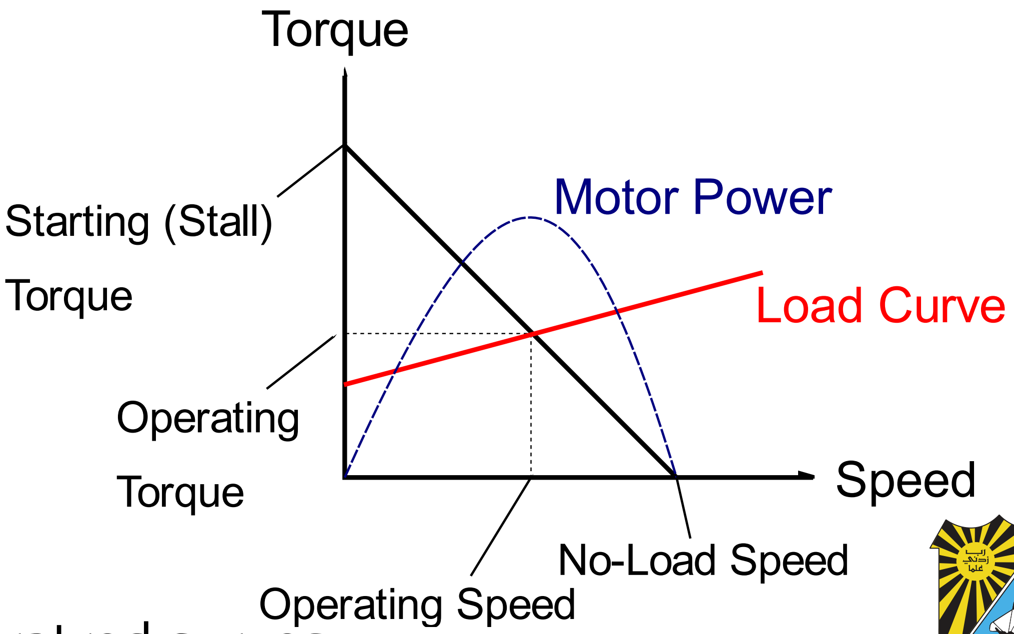
<u>Ideal</u> Steady-State Torque-Speed Curve for a Brushed DC Motor



Note: If you search for Torque-Speed curves you will get different shaped curves, those are for different types of motors or different assumptions made or different operating conditions, but the key characteristics above apply.

- From equation 4, if we consider the steady-state response of the motor, we can simplify by setting inductance $L_a = 0$, we get
  - $\frac{R_a}{K_t} T_m(s) + K_B s \Theta_m(s) = E_a(s) \quad (eq. 6)$
- Taking $\mathcal{L}^{-1}(eq. 6)$ and rearranging
  - $T_m(t) = -\frac{K_B K_t}{R_a} \omega_m(t) + \frac{K_t}{R_a} e_a(t) \quad (eq. 7)$
  - Equation 7 matches the Torque-Speed Curve
- Stall is when $\omega_m = 0$
  - $T_m(t) = \frac{K_t}{R_a} e_a(t)$
- No Load Speed is when $T_m(t) = 0$,
  - $\omega_{no\ load} = \frac{e_a}{K_B}$
- The constants can then be computed from the dyno generatred curves.

Ideal Steady-State Torque-Speed Curve for a Brushed DC Motor



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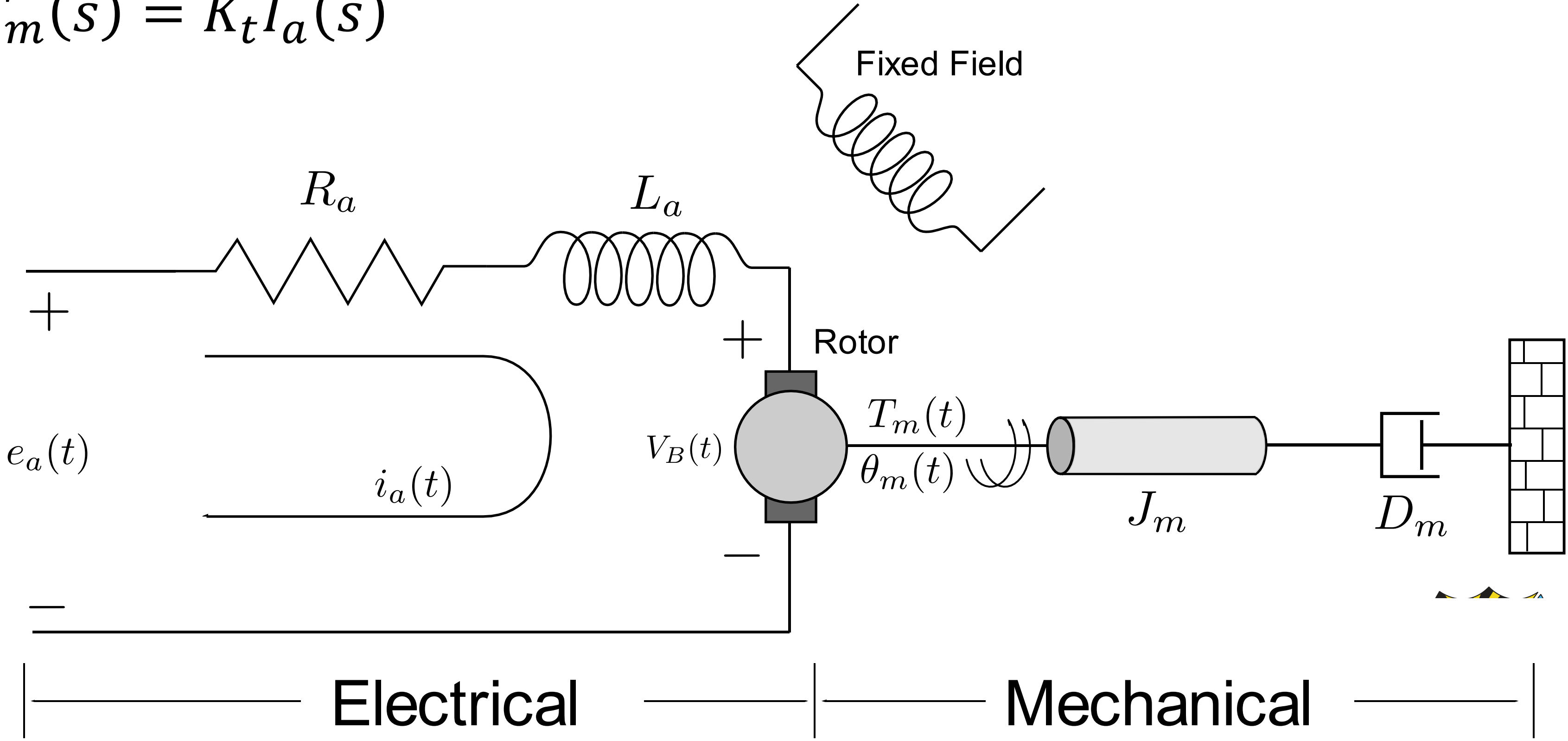
- DC Motor Model, Combining:
  - Electrical Part Gives: $R_a I_a(s) + L_a s I_a(s) + V_B(s) = E_a(s)$
  - Mechanical Part Gives: $T_m(s) = (J_m s^2 + D_m s)\Theta_m(s)$
  - Additional relationship 1: $V_B(s) = K_B s \Theta_m(s)$
  - Additional relationship 2: $T_m(s) = K_t I_a(s)$
- We get:

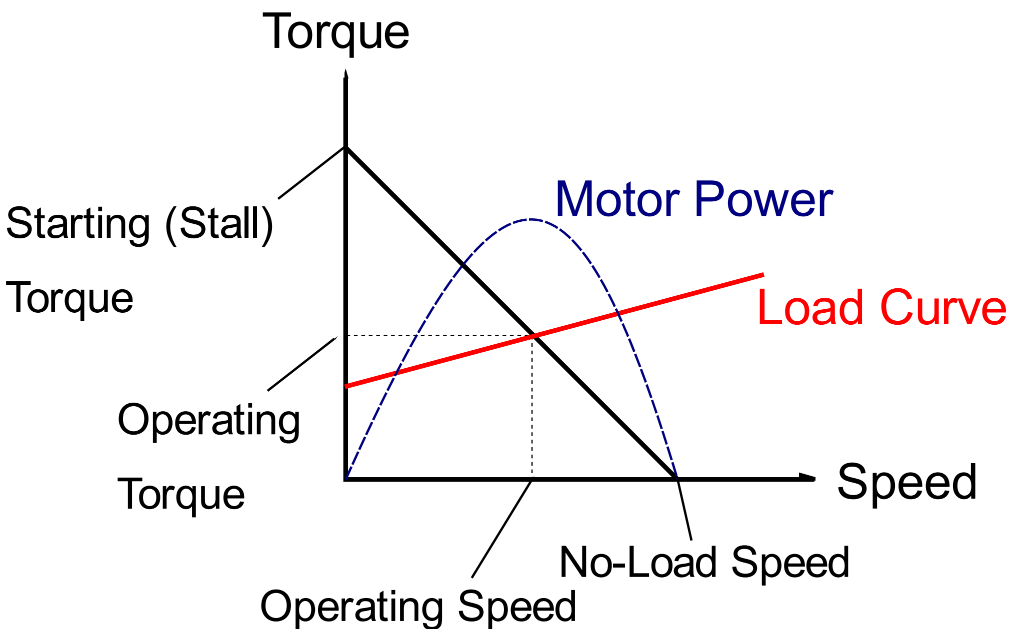
$$\frac{\Theta_m(s)}{E_m(s)} = \frac{K_t/(R_a J_m)}{s[s+\frac{1}{J_m}(D_m+\frac{K_t K_B}{R_a})]}$$



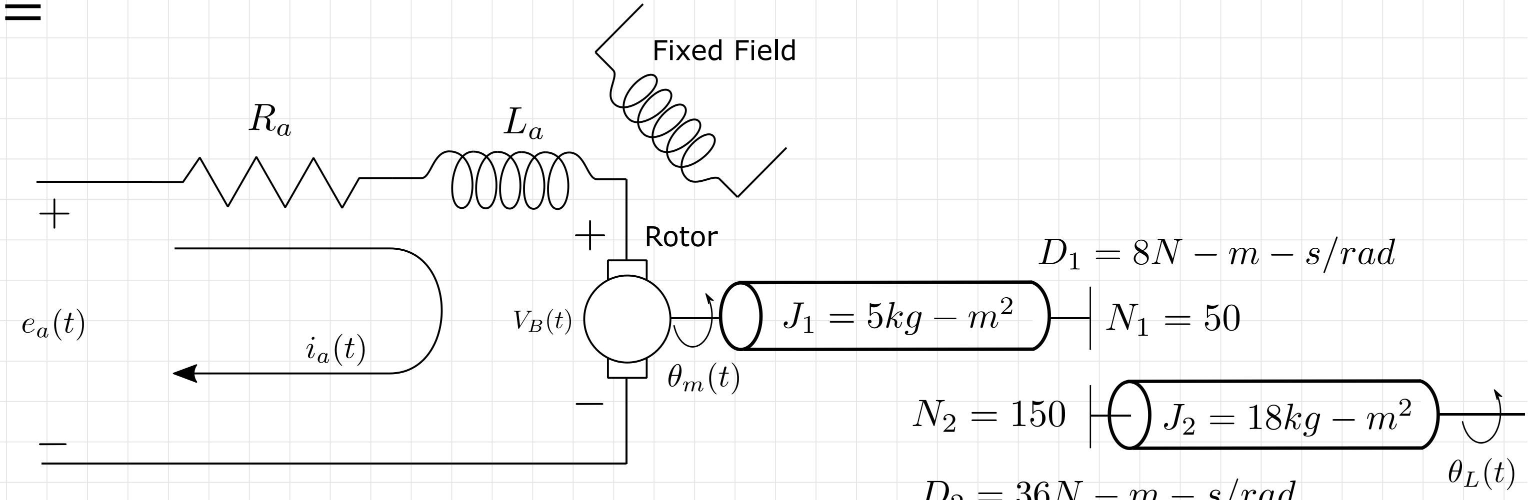
- Torque-Speed Curve

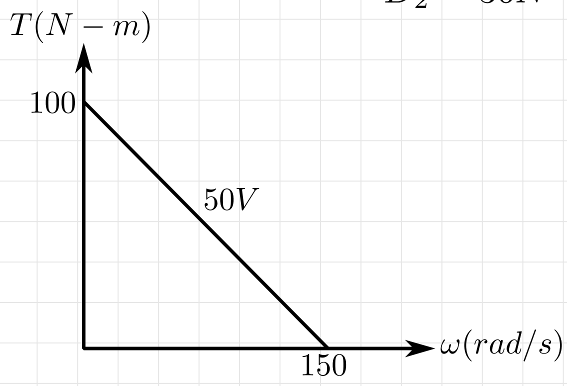
$$T_m(t) = -\frac{K_B K_t}{R_a}\omega_m(t) + \frac{K_t}{R_a}e_a(t)$$

<u>Ideal</u> Steady-State Torque-Speed Curve for a Brushed DC Motor



For the motor, load, and torque-speed curve shown. Find the transfer function, $G(s) = \frac{\Theta_L(s)}{E_a(s)}$. Nise: 2-43

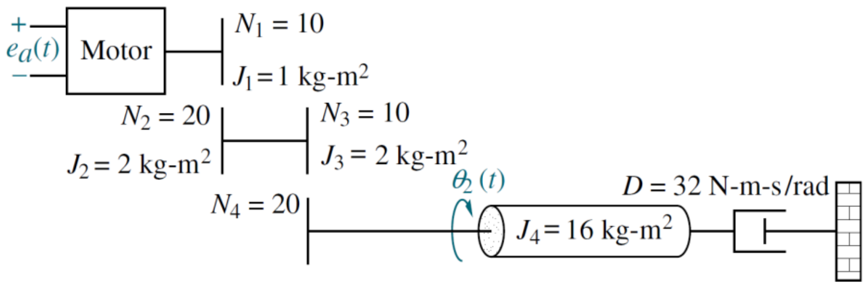


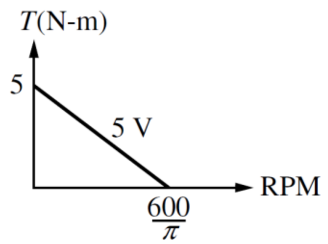


In this chapter, we derived the transfer function of a dc motor relating the angular displacement output to the armature voltage input. Often, we want to control the output torque rather than the displacement. Derive the transfer function of the motor that relates output torque to input armature voltage.

The motor whose torque-speed characteristics are shown in the figure drives the load shown in the diagram. Some of the gears have inertia. Find the

transfer function, $G(s) = \frac{\Theta_2(s)}{E_a(s)}$. Nise: 2-44





Practice Problem 2

A dc motor develops 55 $N-m$ of torque at a speed of $600 rad/s$ when $12 volts$ are applied. It stalls out at this voltage with $100 N-m$ of torque. If the inertia and damping of the armature are $7 kg-m^2$ and $3 N-m-s/rad$, respectively, find the transfer function, $G(s)=\frac{\Theta_L(s)}{E_a(s)}$, of this motor if it drives an inertia load of $105 kg-m^2$ through a gear train, as shown in the figure. Nise: 2-45