Kuwait University
College of Engineering and Petroleum

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ME417 CONTROL OF MECHANICAL SYSTEMS

Part II: Controller Design Using Root-Locus

Lecture 5: Improving Transient Response

Spring 2021

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- Objectives:
  - Explore the use of ideal derivative compensators to improve transient response
  - Explore the use of a lead compensator to improve transient response
- Reading:
  - *Nise: 9.3*
- Practice problems included

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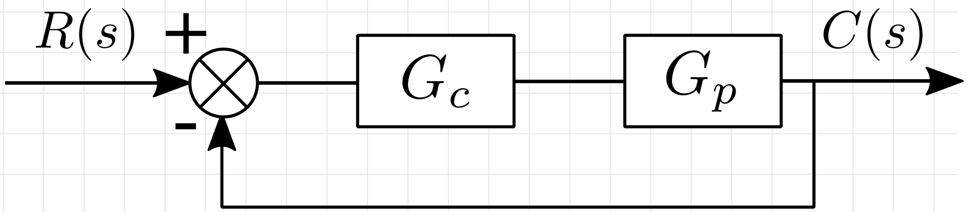
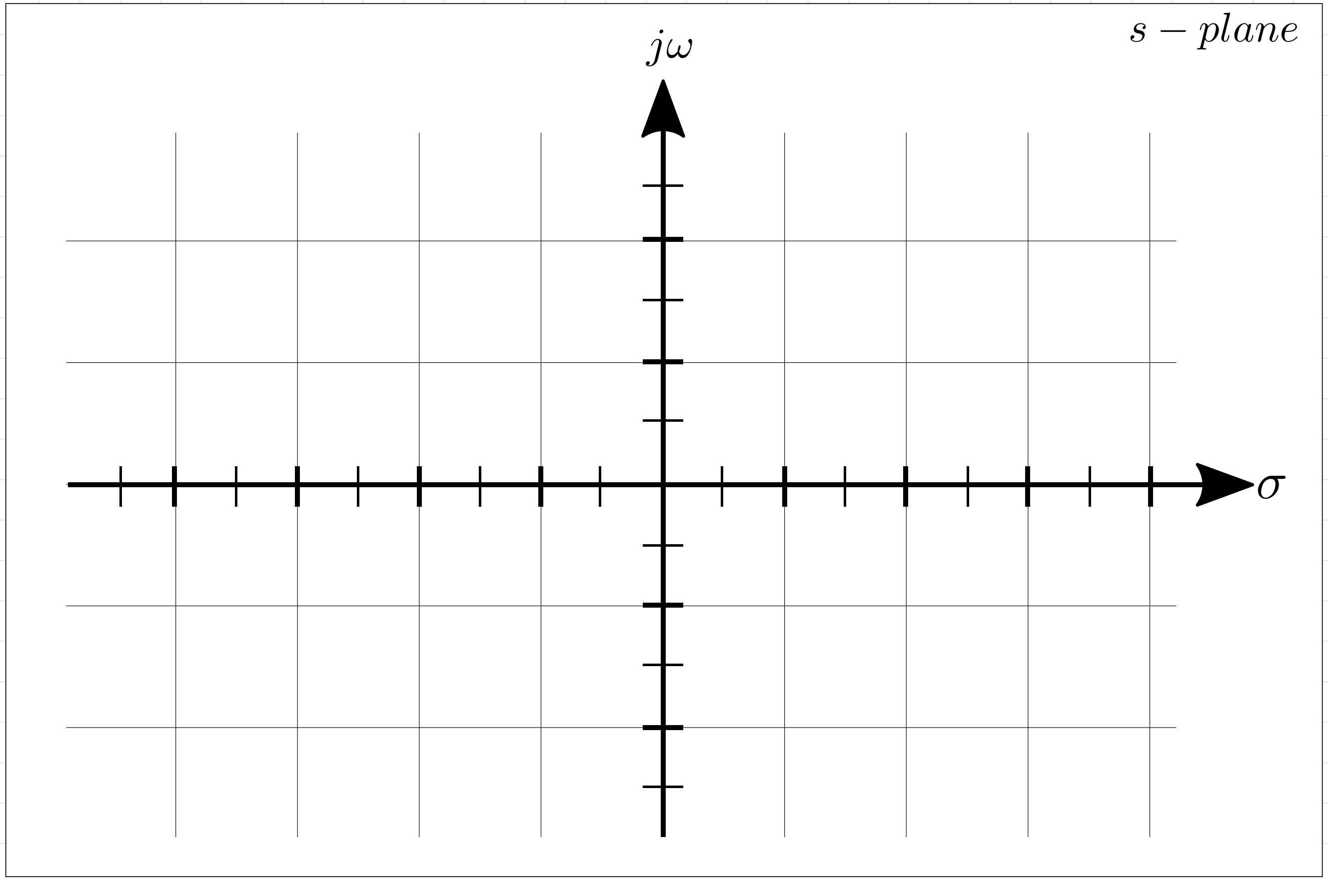
Find the real axis locations of the closed-loop system's poles and zeros given the following:

$$G_c = 1, \quad G_p = \frac{(s-3)(s-6)}{(s+1)(s+4)}$$



Is the closed-loop system stable at this point?

Find the imaginary component of the CL poles.



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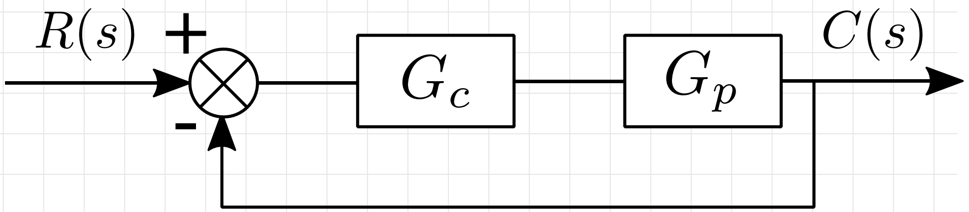
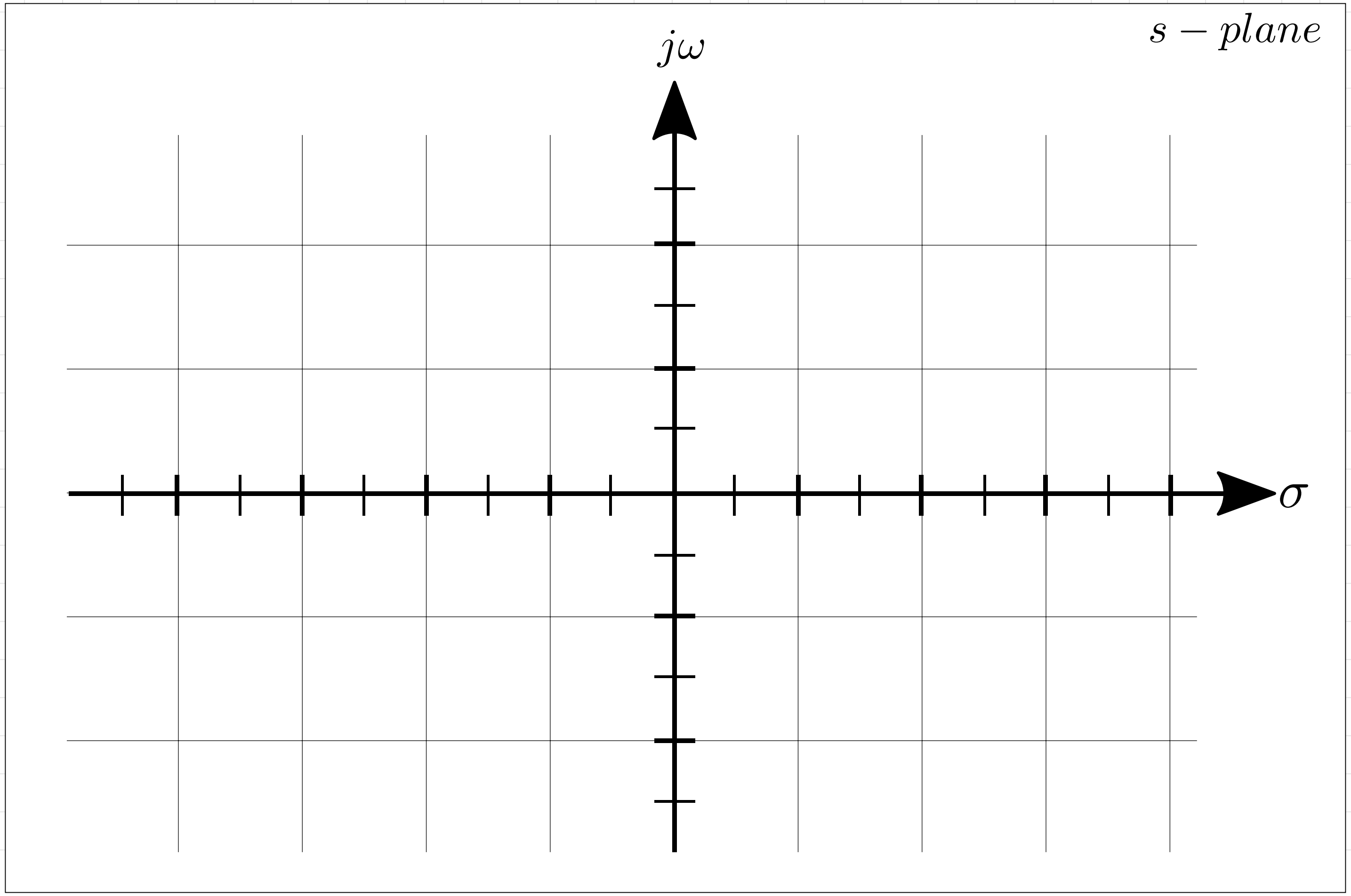
Draw the root-locus of the system with $G_c = K$, then with a choice of an additional compensator poles and/or zeros, stabilize the system for all gains, and make the complex root-locus intersect the $T_s = 1s$ line.



$$G_p = \frac{1}{(s+10)(s+2s+2)}$$



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Improving Transient Response via Compensation

- What if the feedback system is unstable for some value of the proportional gain, and we would like to stabilize the system for all, or at least a higher range, of gain values?

- What if the performance we seek requires the dominant closed-loop poles to be placed outside the root-locus achieved with just a proportional or proportional-integral controllers

- We can change the location of the root-locus by adding appropriate compensators

    - *Changing the shape of the root-locus means changing the possible locations of the closed-loop poles, thus affecting stability/transient response.*

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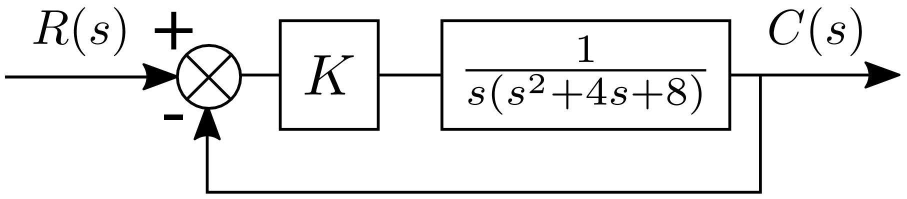
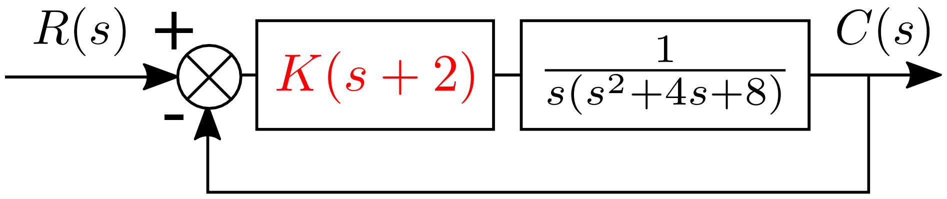
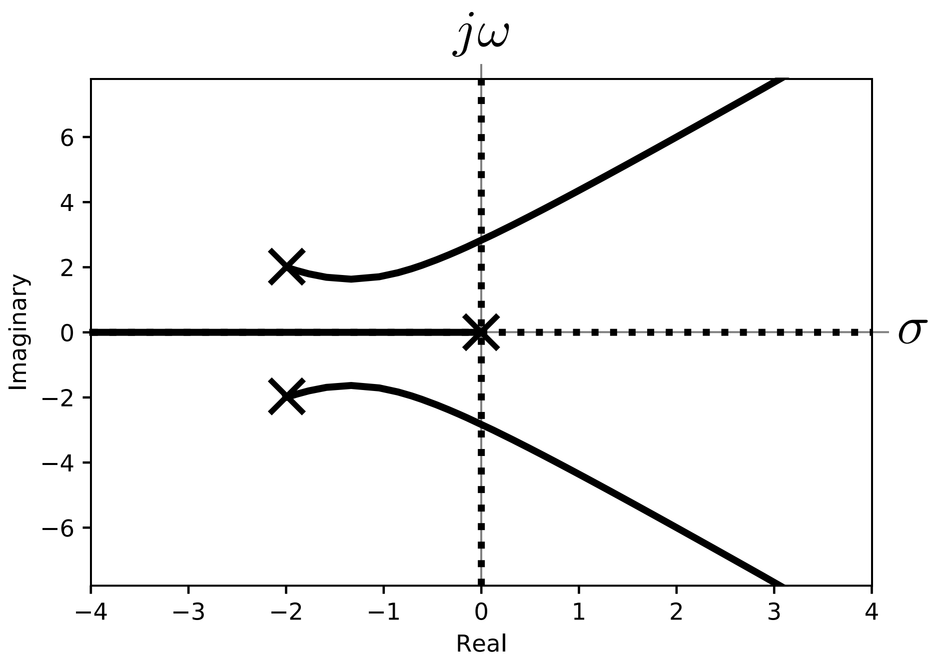
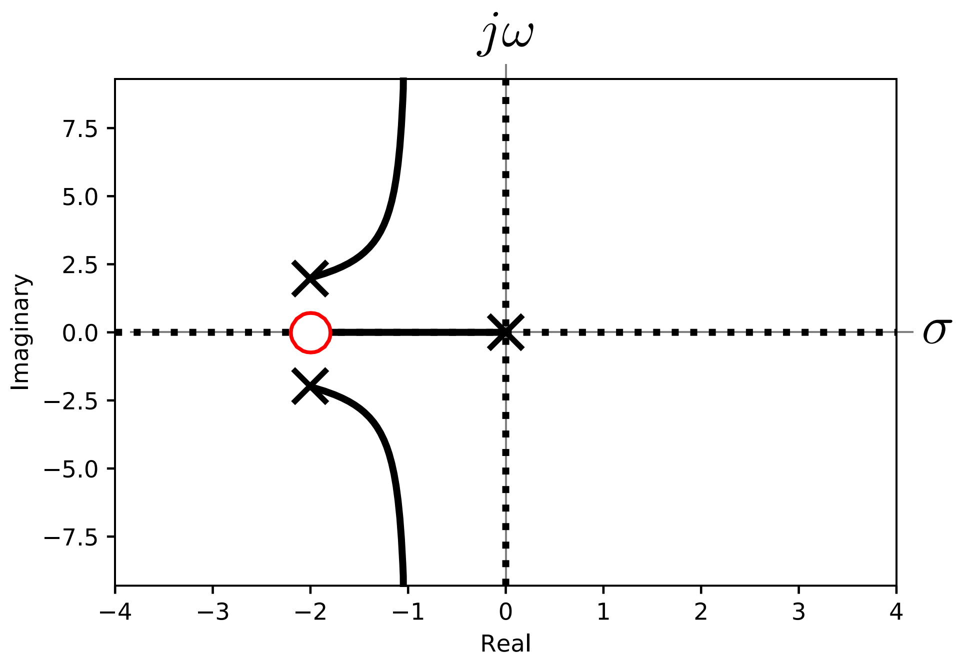
- A compensator can make a possibly unstable system, stable for all gain values.



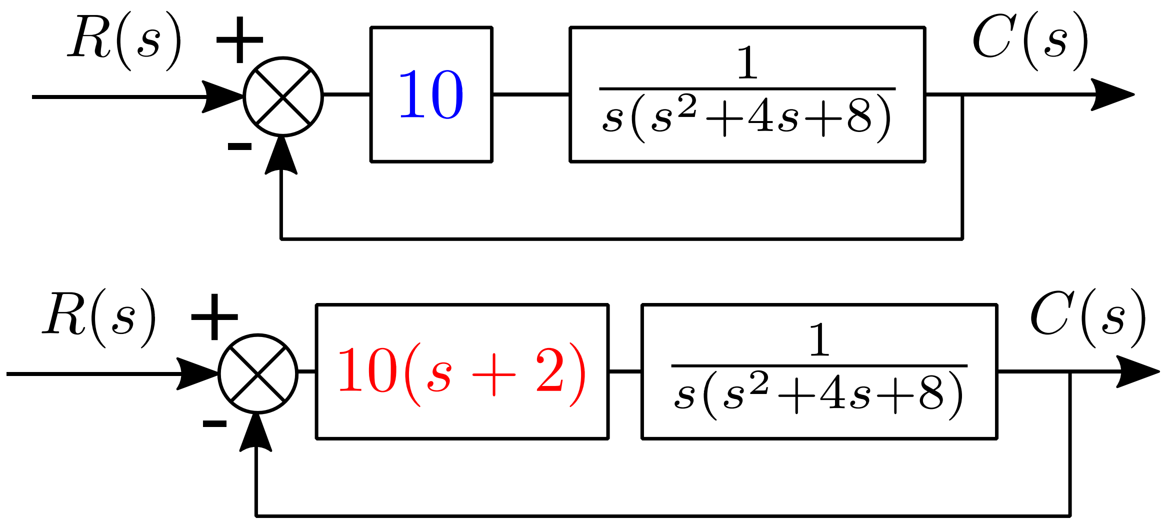


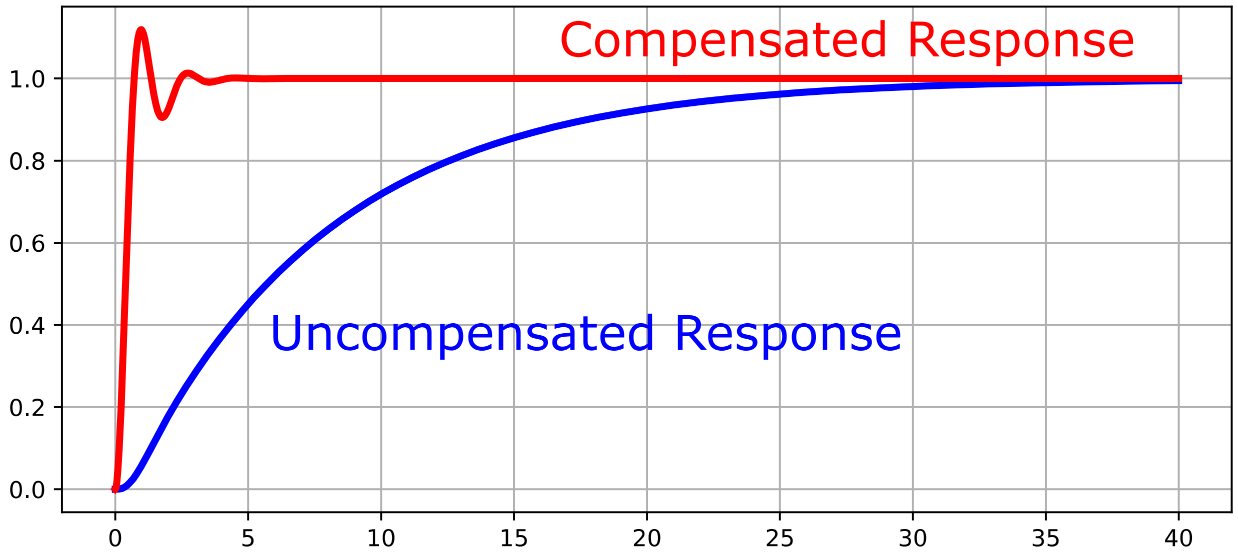




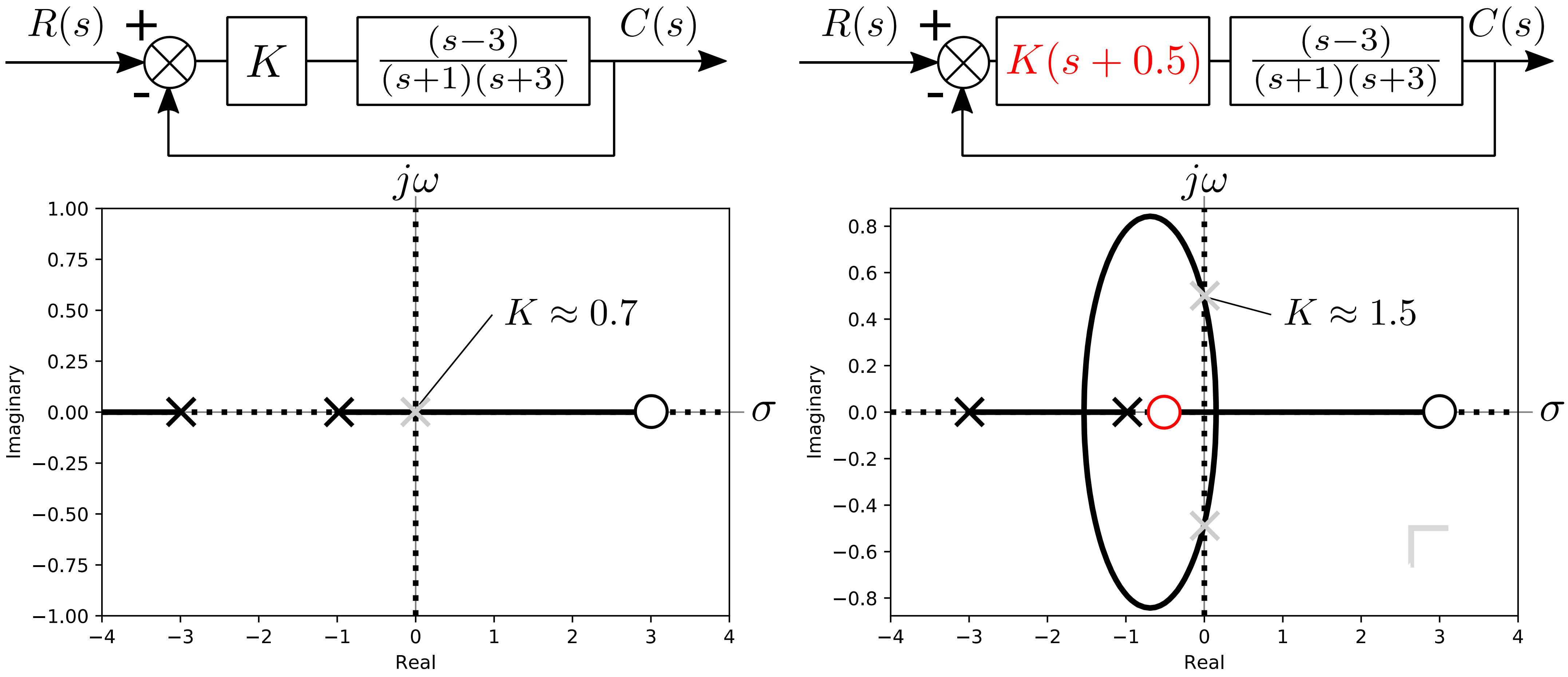
Improving Transient Response via Compensation

- Compensation can increase the speed of response

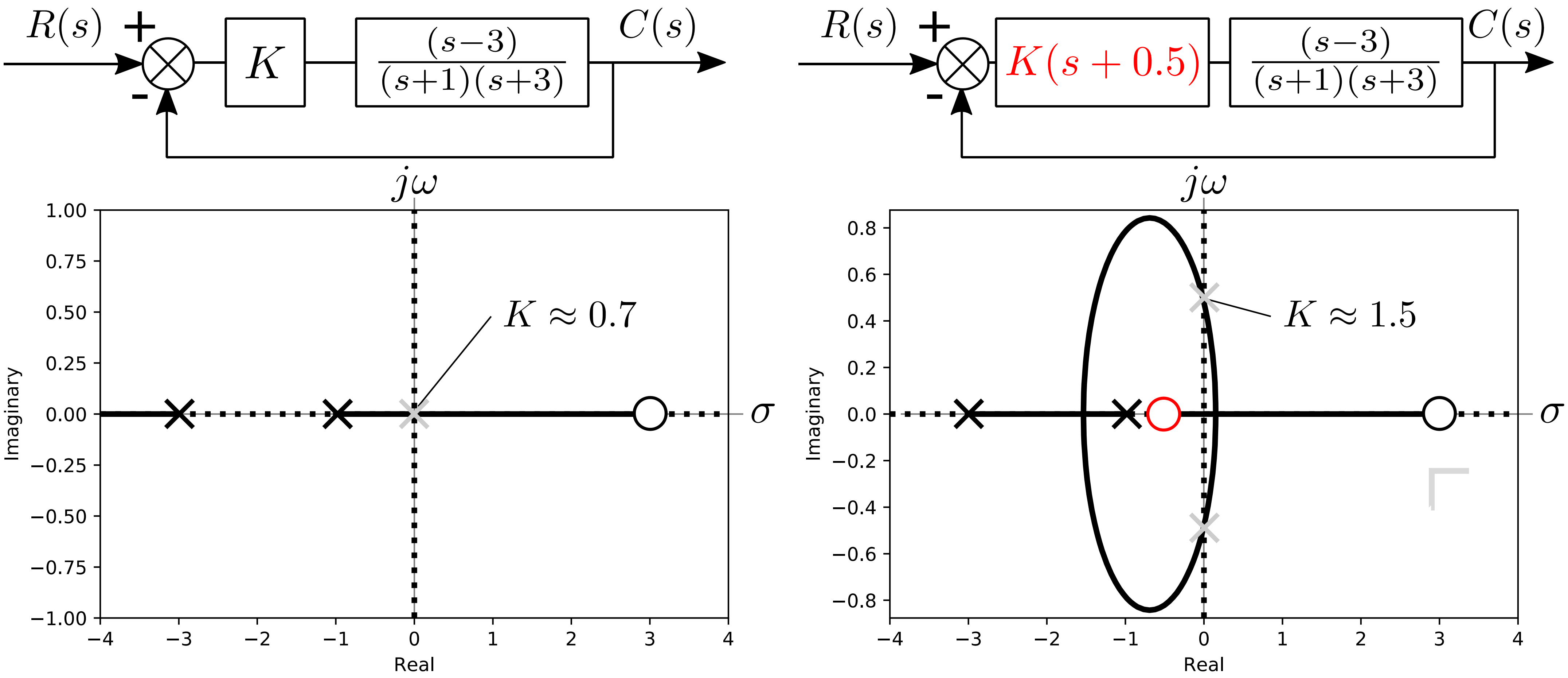




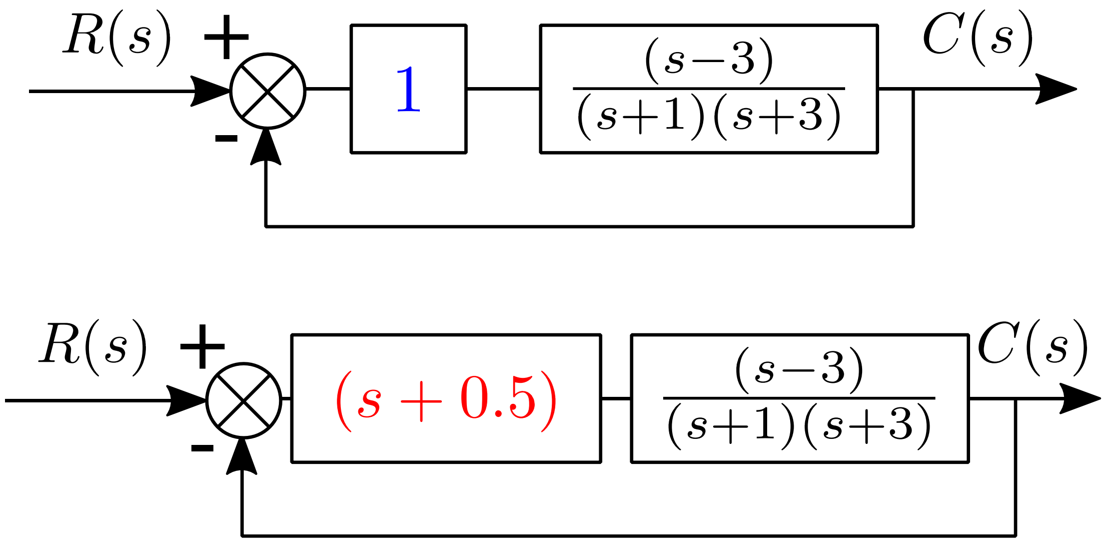
- A compensator can increase the range of the gain for which the system is stable.

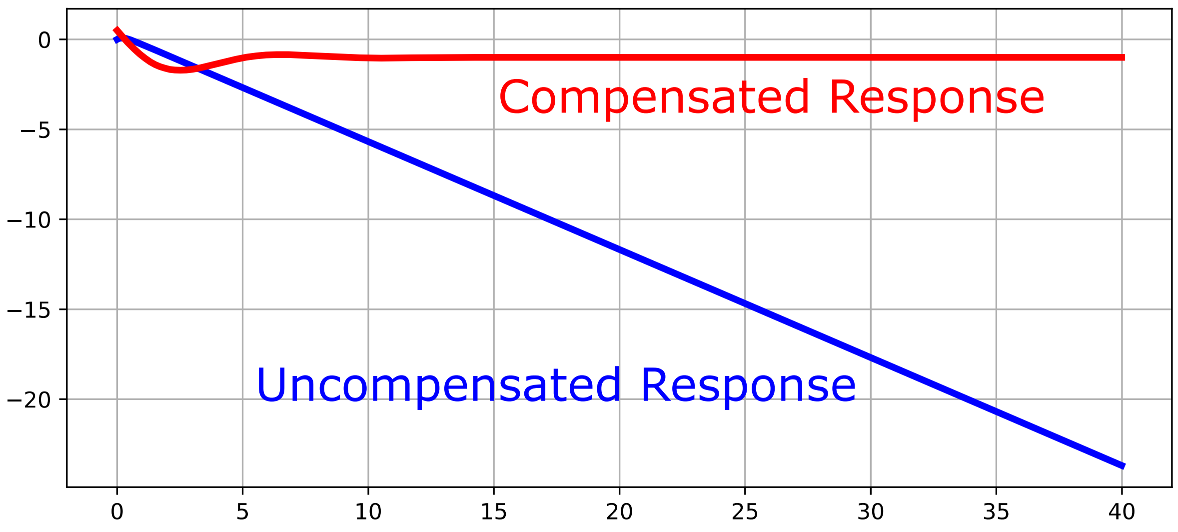


- A compensator can change the possible locations of the closed-loop poles for varying K
  - By changing the shape of the root-locus.



- For the same gain the system is stabilized with a compensator
  - *Same gain does not mean same controller output range* $u(t)$





Improving Transient Response via Compensation

- We will introduce two compensators that are used to improve transient response.
    1. *Ideal Derivative Compensator (a.k.a PD: Proportional-Derivative Controller)*
    2. *Lead Compensator*
- *Remember that in the world of control, we are not limited to the above-mentioned compensators, but they are widely used, nevertheless.*
- *The more complex the compensators, the more it is hard to implement them practically in a real control system.*

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Ideal Derivative Compensator (PD Controller)

- The ideal derivative compensator is also known as the Proportional-Derivative Controller. It seeks to:
  - Improve the transient response in the form of
    - *Stabilization*
    - *Improve Settling Time*
    - *Reduce Over-shoot*
      - *Misplacement of the derivative zero can worsen the response*
  - Places a zero on the real axis, in the LHP.
  - Requires active components to implement
  - Relies on differentiating the error signal
  - *This can be unreliable for noise signals or signals with low sampling rate.*

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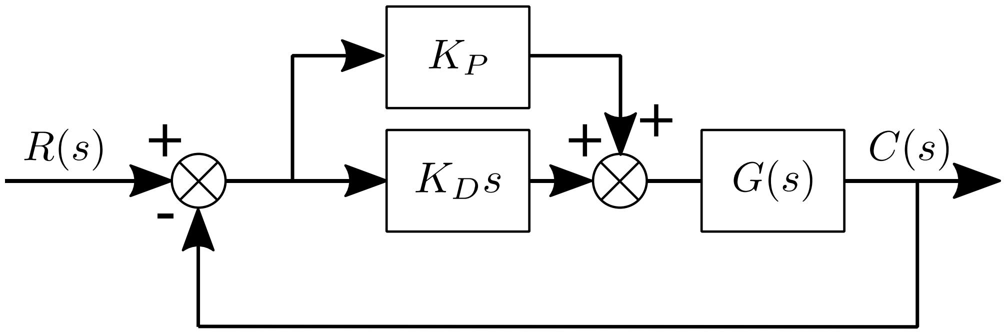
Ideal Derivative Compensator (PD Controller)

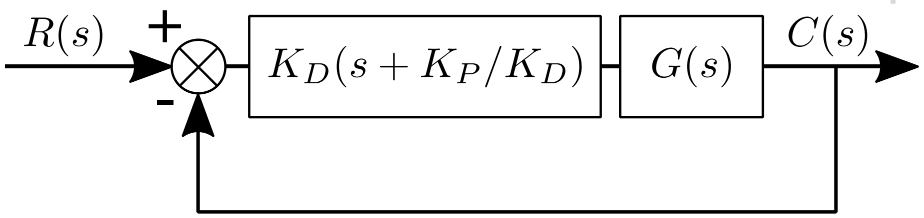
- The Ideal Derivative Compensator is of the form

$$G_c = K(s+z) = K_p + K_D s = K_D(s + K_P/K_D) = \frac{K_P}{T_D}(s + T_D)$$

Where $T_D = K_P/K_D$

- *Note that with **PI** Control, the initial choice of K for the feedback system wasn't affected much by the addition of the integral controller.* $K \approx K_P$
- *As long as the PI's zero location was close to the PI's pole*
- But with **PD** Control, *the feedback gain is equal to the derivative gain,* $K = K_D$





The introduction of the zero by the PD Controller:

- Reduces the number of asymptotes; reducing the possibility that the root-locus extends to the RHP.
  - *E.g: Instead of three asymptotes:* $\frac{\pi}{3}, \frac{3\pi}{3}, \frac{5\pi}{3}$, *get two asymptotes:* $\frac{\pi}{2}, \frac{3\pi}{2}$
- Can reduce the affect of the slow decaying poles (low $\zeta\omega_n$ values), increasing the response rate of the system (reducing settling time).
  - *E.g: for* $G_p = \frac{1}{(s+0.1)(s^2+4s+8)}$, *the compensator* $G_c = (s+0.2)$, *reduces the effect of the pole at* 0.1
- Can increase damping by "pulling" the root locus from the complex space and toward the real axis (lowering θ, increasing $\zeta = cos\theta$)

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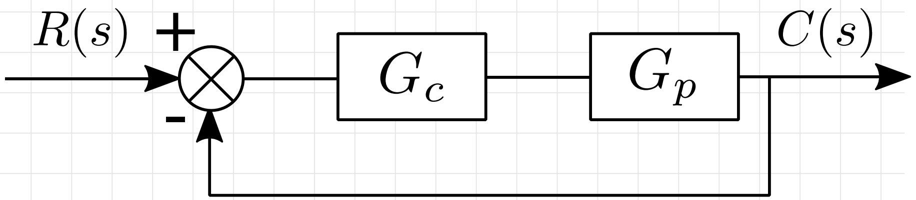
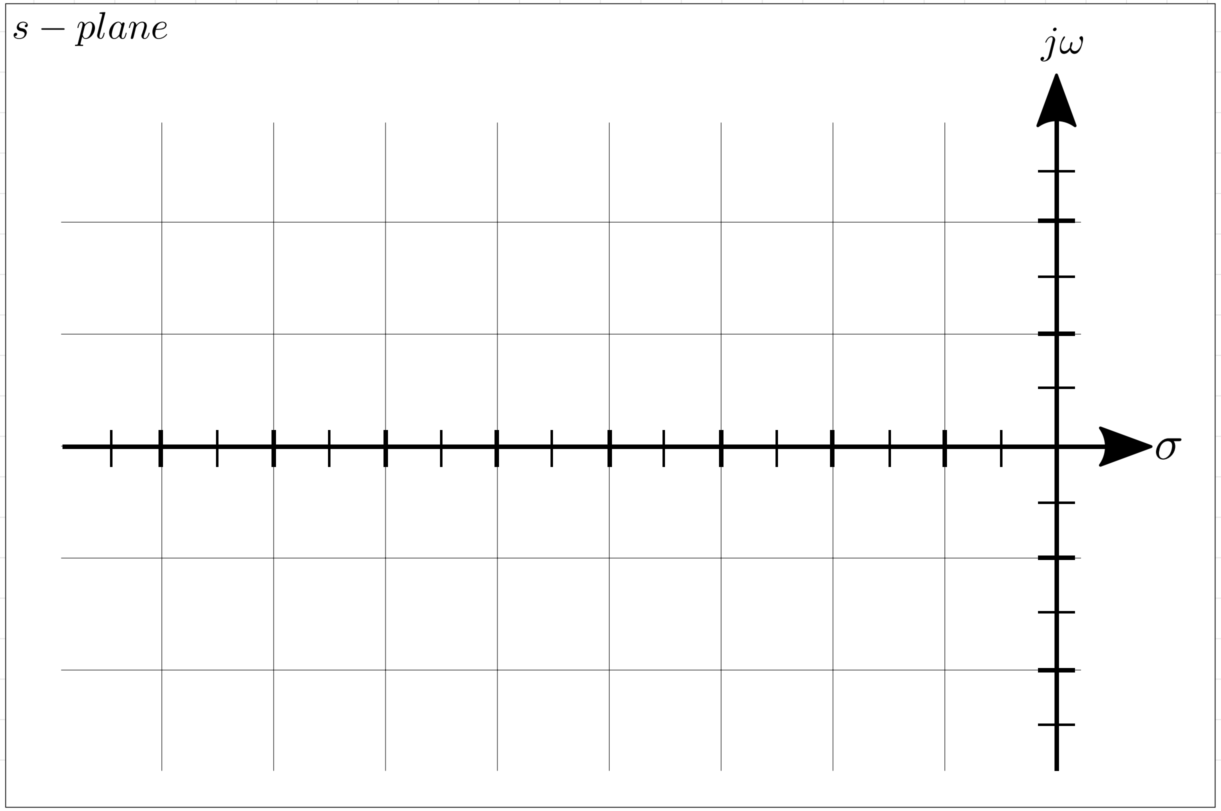
Show, by computing the closed-loop transfer function, that the addition of the Ideal Derivative Compensator, increases the speed of the response of the system. Show again, using the root-locus.



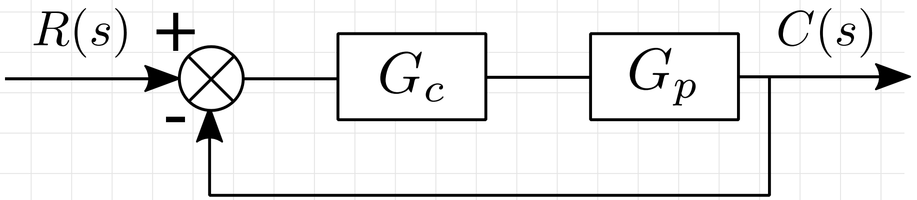
$$G_p = \frac{1}{(s^2+2s+2)}, G_{C\,uncomp} = 2, G_{C\,comp} = 2(s+1)$$

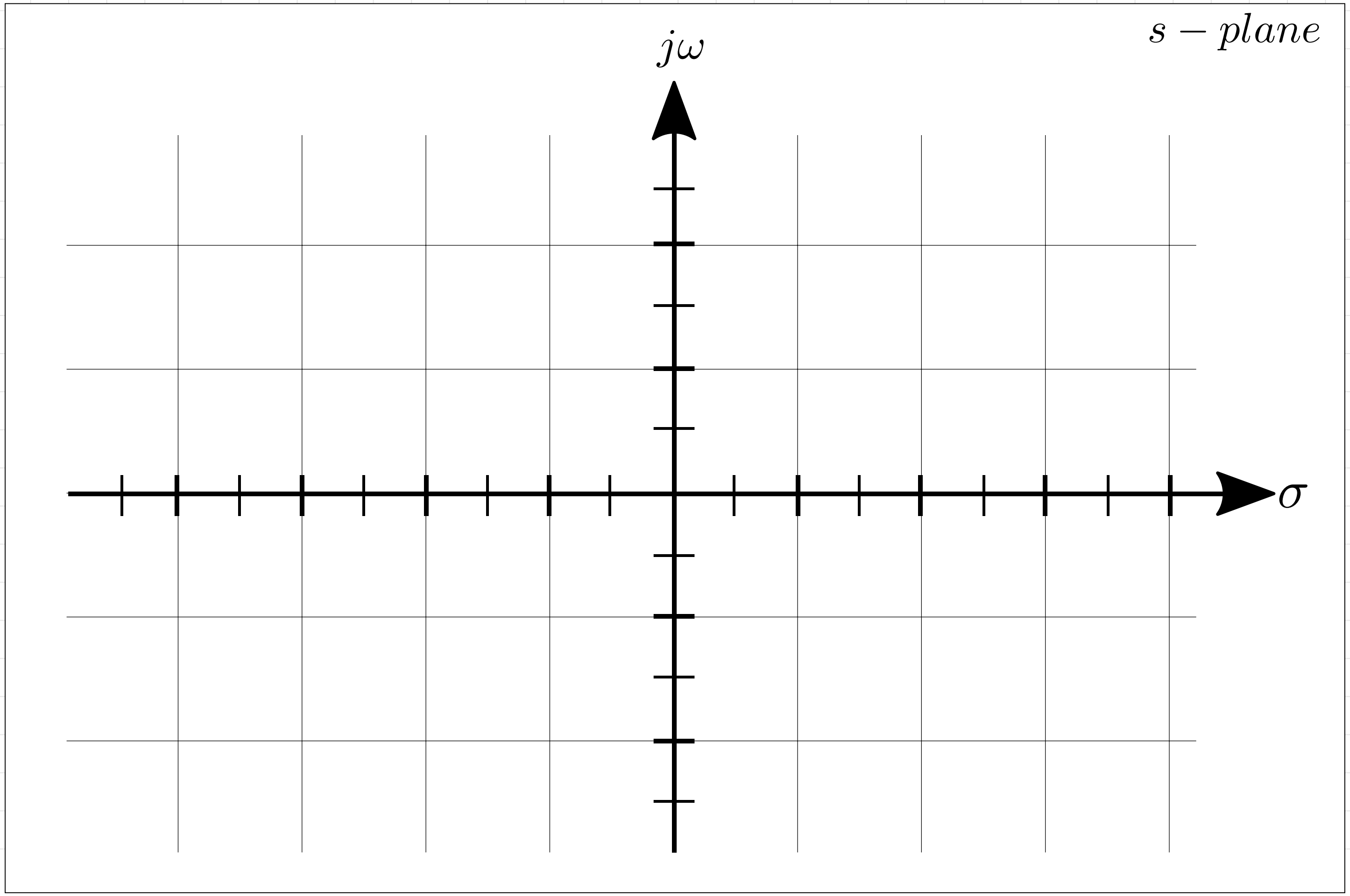


Continue Example

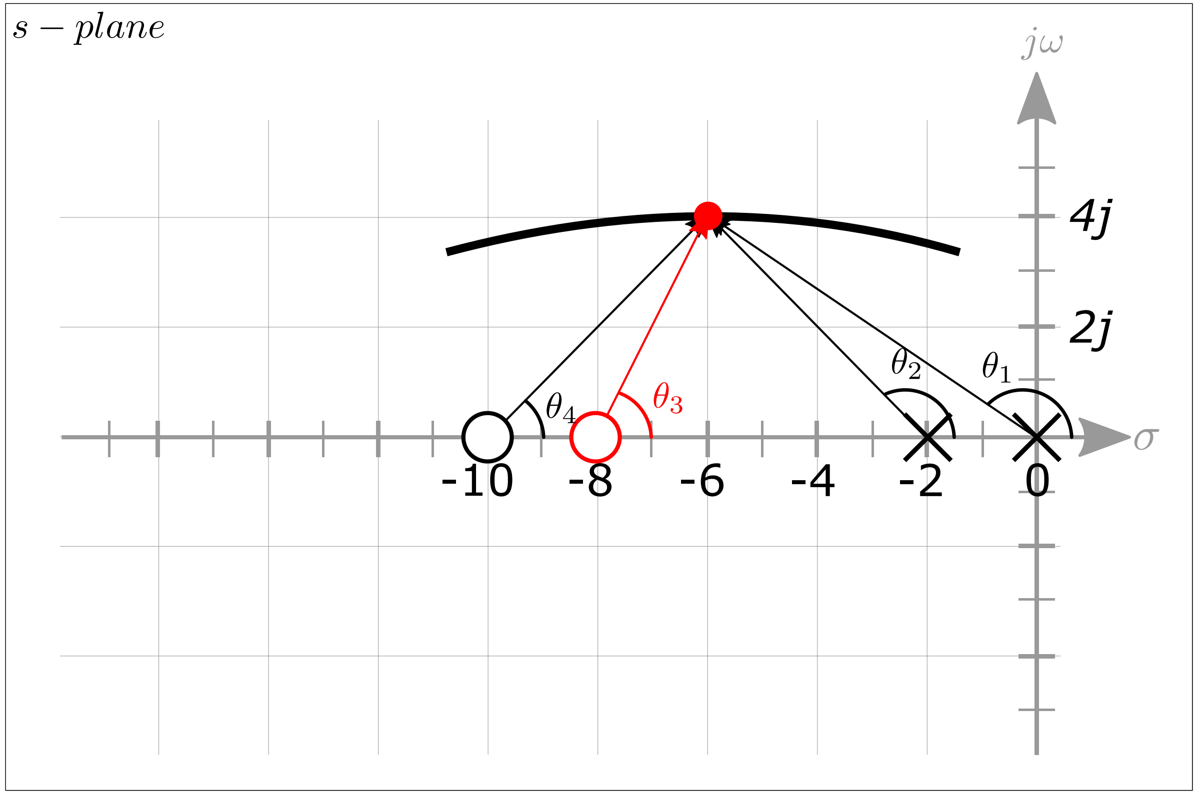
Design a controller that stabilizes the system and reduces the settling time of the closed-loop system to $T_s = 0.5s$

$$G_p = \frac{1}{(s-1)(s-2)}$$



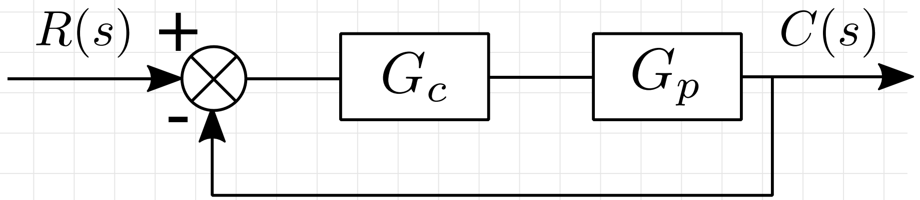


- We can use the angle condition to place the compensator pole in order to produce the desired closed-loop pole location.
- Angle condition
- $\angle KG(s) = \angle\sum\theta_{zeros} - \angle\sum\theta_{poles} = (2k+1)180^o$
- Example:
- $\theta_2 = 180 + \theta_1 + \theta_2 - \theta_3$
- $\theta_2 = \tan^{-1}(\frac{4}{-6-z})$

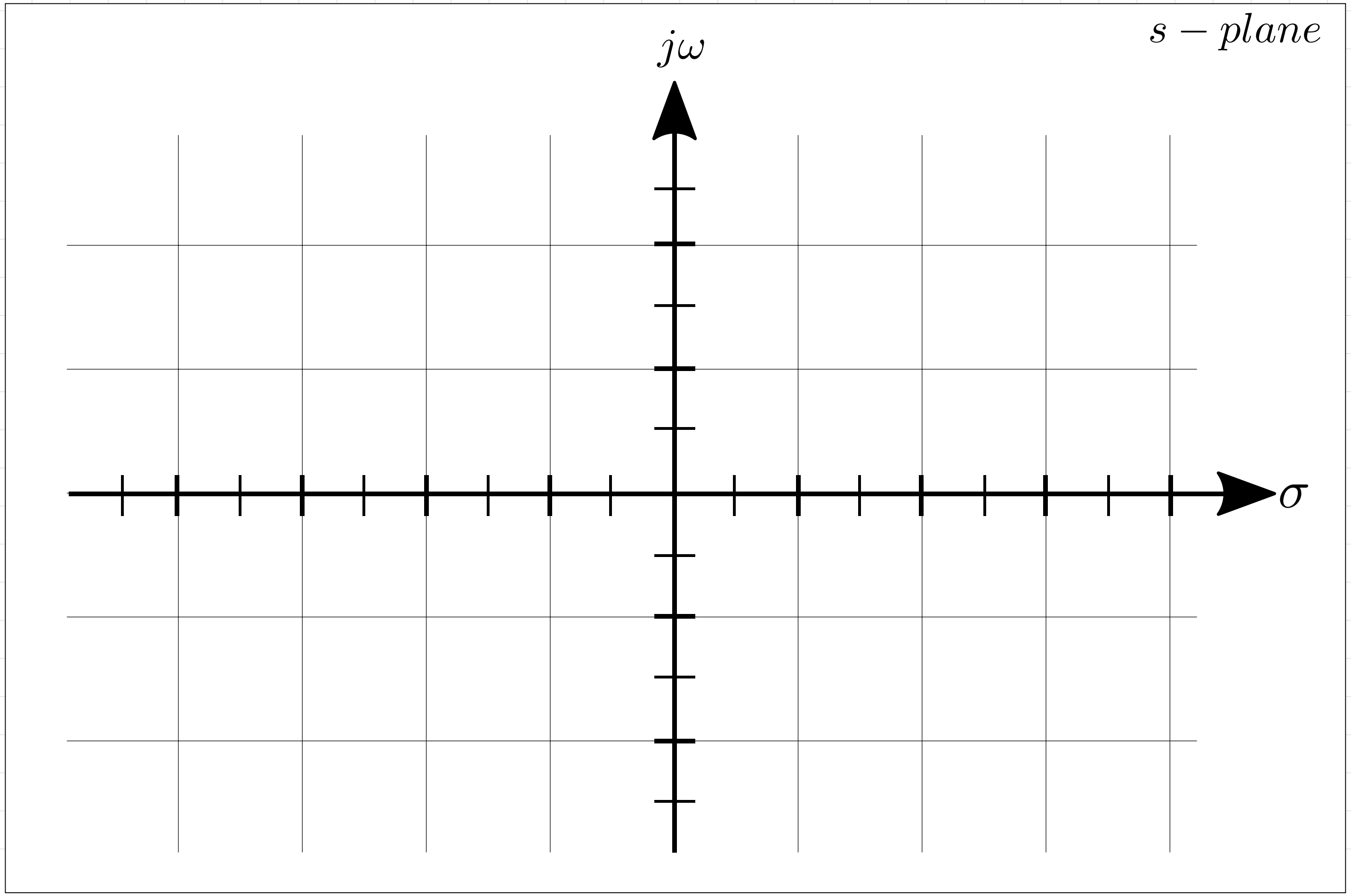


Design a controller that reduces the settling time of the closed-loop system to $T_s = .5s$ with a damping ratio of $\zeta = 0.866$

$$G_p = \frac{1}{(s-1)(s-2)}$$



Do you think the response will have a settling time of exactly $0.5s$? Why?



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Lead Compensator

- The purpose of the Lead Compensator and Ideal Derivative Compensator (PD Controller) are the same, they both seek to improve transient response
- The Lead Compensator doesn't require active circuits
  - Can be implemented using passive circuits
    - *This is more relevant in electrical control systems*
    - *There are analogous passive lag and lead mechanical compensators*
- The Lead Compensator reduces the affect of noise from the error derivative signal, compared to the PD Controller.
  - *The compensator pole reduces the affect of the compensator zero*
- It's of the form

$$G_c = K\frac{s+z}{s+p}$$

  - Where $\theta_z - \theta_p = \theta_c$ is the angle contribution of the compensator that would place the closed-loop pole in the desired location.

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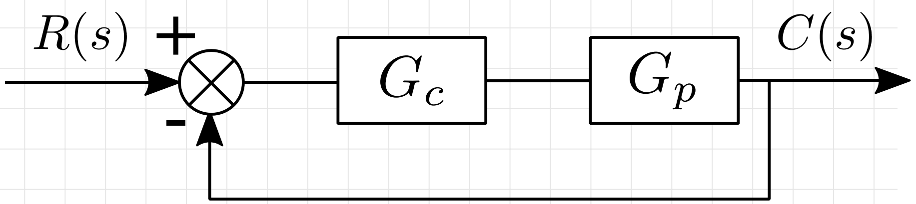
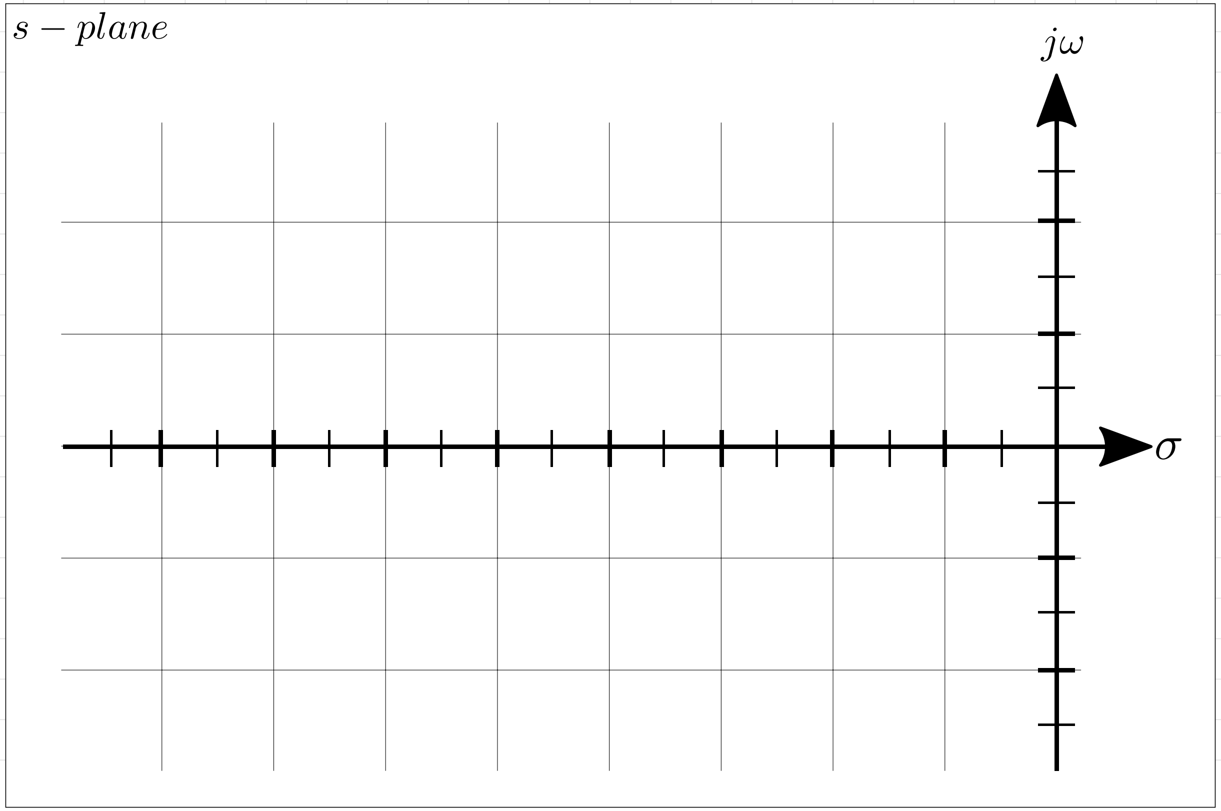
Design a Lead Compensator, that would place the closed-loop poles at $s = -5 \pm 5j$. Choose the zero of the compensator to be at -1.

Is a second-order approximation valid for the closed-loop system?

$$G_p = \frac{1}{(s+1)(s+2)}$$





Nise 6th Global Edition:

9-8, 9-9, 9-11, 9-14, 9-16